(9.8) ACTUALLY DOING THE CALCULATION (SUBSTITUTING (9.7) INTO (9.3)) GIVES,

$$0.25 \leq \Omega_0 h^{1/2} \leq 0.40$$

BUT $\Omega_0 \cong 0.3 h^{-1/2}$ IS WITHIN THE RANGE.

(11.5) NOTE $1/24{,}000 \approx 4.15 \times 10^{-5}$

(11.6) $\dfrac{T(t)}{T_0} = \left(\dfrac{t}{t_0}\right)^{-2/3}$ $\quad t_0 = 4 \times 10^{17}\,\text{sec} = 13.7 \times 10^9\,\text{yr}$

(11.7) $a(t) = \left(\dfrac{t}{t_0}\right)^{2/3}$ $\quad (5.15) \Rightarrow \dfrac{T(t_{eq})}{T_0} = \dfrac{1}{a_{eq}}$

(11.8) $t_{eq} = \left(\dfrac{T_0}{T_{eq}}\right)^{3/2} t_0 = \left(\dfrac{2.725}{66{,}000}\,\Omega_0^{-1} h^{-2}\right)^{3/2} \times 4 \times 10^{17} = 1.1 \times 10^{11}\,\Omega_0^{-3/2} h^{-3}\,\text{sec}$

(11.11) FROM EQN (10.2)

(11.12) $H = \dfrac{\dot{a}}{a}\,;\ a \propto t^{1/2} \Rightarrow \dot{a} \propto \frac{1}{2} t^{-1/2} \Rightarrow H = \dfrac{\frac{1}{2} t^{-1/2}}{t^{1/2}} = 1/2t$

$$\left(\frac{1}{2t}\right)^2 = \frac{1.68 \times 8\pi G}{3} \times \frac{\alpha T^4}{c^2} \Rightarrow t^{-1/2} = \left(\frac{4 \times 1.68 \times 8\pi G \times \alpha}{3c^2}\right)^{1/4} T$$

$$t^{-1/2} = \left(3160 \times 10^{-44}\,\text{sec}^{-2}\,K^{-4}\right)^{1/4} T = \left(7.5 \times 10^{-11}\,\text{sec}^{-1/2}\,K^{-1}\right) T$$

$$t^{-1/2} = \frac{T}{1.3 \times 10^{10}\,K\,\text{sec}^{1/2}}$$ TAKING $K_B = 8.62 \times 10^{-11}\,\text{MeV}\,K^{-1}$

$$t^{-1/2} = \frac{K_B T}{1.1\,\text{MeV}}$$

(12.10) $Y_4 = \frac{\text{TOTAL MASS } {}^4He}{\text{TOTAL MASS UNIVERSE}} = \frac{\text{TOTAL MASS } {}^4He}{\text{TOTAL MASS } {}^4He + \text{TOTAL MASS } H}$

$m_{4He} = 4m_p = 4m_H = 4m_n$

$N_{4He} = N_n/2$ $\quad N_p = N_H + 2N_{4He} = N_H + 2\left(\frac{N_n}{2}\right) = N_H + N_n$

$N_H = N_p - N_n$

$$Y_4 = \frac{m_{4He} N_{4He}}{m_{4He} N_{4He} + m_H N_H} = \frac{4m_p \cdot \frac{N_n}{2}}{4m_p \cdot \frac{N_n}{2} + m_p(N_p - N_n)}$$

$$= \frac{2N_n}{2N_n + N_p - N_n} = \frac{2N_n}{N_n + N_p} = \frac{2}{1 + N_p/N_n} = \frac{2}{1 + 7.3} = 0.24$$

(13.2) $(5.19) \rightarrow a(t) \propto t^{1/2} \rightarrow \dot{a}(t) \propto t^{-1/2} \rightarrow a^2 H^2 = a^2\left(\frac{\dot{a}^2}{a^2}\right) = \dot{a}^2 \propto t^{-1}$

(13.3) $(5.15) \rightarrow a(t) \propto t^{2/3} \rightarrow \dot{a}(t) \propto t^{-1/3} \rightarrow a^2 H^2 = \dot{a}^2 \propto t^{-2/3}$

(13.15) FROM (13.1): $|\Omega_{tot}(t) - 1| \propto \dot{a}^{-2}$ (SINCE $H = \frac{\dot{a}}{a}$)

FROM (13.12): $\dot{a}(t) \propto \exp\left(\sqrt{\frac{\Lambda}{3}}\,t\right)$, SO $\dot{a}^{-2} \propto \exp\left(-\sqrt{\frac{4\Lambda}{3}}\,t\right)$

AND $|\Omega_{tot} - 1| \propto \exp\left(-\sqrt{\frac{4\Lambda}{3}}\,t\right)$

(13.18) $|\Omega_{tot}(t_r) - 1| = \frac{t_r}{t_0}|\Omega_{tot}(t_0) - 1| \leq \frac{10^{-34}}{4\times10^{17}}(0.1) \approx 3\times10^{-53}$

(13.20) $\Delta t = 10^{-34} - 10^{-36} = 100\times10^{-36} - 1\times10^{-36} = 99\times10^{-36}$ sec

$\frac{a_{final}}{a_{initial}} = \exp[H\,\Delta t] = \exp[10^{36}\times 99\times10^{-36}] = e^{99}$

$\ln e^{99} = 99 = \ln(10^b) = \log(10^b)\cdot\log_e 10 = 2.3b$

$b = 99/2.3 = 43$, SO

$e^{99} \approx 10^{43}$