Gravitational Lensing:

Addenda Notes for Chapter Seven

Dave, 1/20/20 – 2/8/20

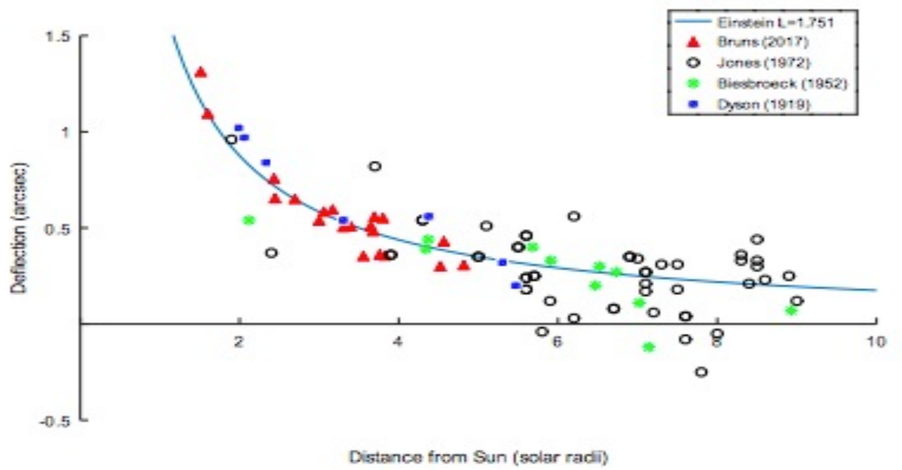

Figure 1: Solar Eclipse Data: "A Century of Light-Bending Measurements," "https://arxiv.org/pdf/2002.01179.pdf

In this chapter, the topics of bending of starlight, gravitational lensing, and the time delay of radar (Shapiro delay) are discussed as approximations for weak gravitational fields, $GM/c^2r \ll 1$. **The weak field metric (Eqn. 7.33)** can be derived from just "The Principle of Equivalence" and Special Relativity not really needing General Relativity , e.g., see Schiff (Am. J. Phys. 1960) :
http://eotvos.dm.unipi.it/documents/SchiffDickeEtAlPapers/Schiff1960AJPSchiff-2.pdf

{A separate outline of this is in my Book 2 in the essay "Learning Quantum Mechanics and Relativity," pages 47-50, http://www.sackett.net/DP_Stroll2.pdf }

Light bending angle ($\Delta\theta = 4GM/bc^2$) and Shapiro delay depend on gravitational space-time curvature with equal contributions of "curvature of time" and the "curvature of space." Gravitational redshift and ordinary Usual Newtonian gravitational effects are only due to time curvature ($dt/d\tau$ {dee-tee-dee-tau}) which can only give half of the correct answer for light deflection. Very rapidly moving particles above Newtonian speeds increasingly see space curvature as well. The standard pictures of space being distorted by a heavy ball on a rubber sheet and causing Newtonian planets to orbit – that should really represent time curvature as a function of radial distance from the ball.

HISTORY:

Einstein's calculation of gravitational lensing by stars predicted very slight bending that was hard to measure (see Fig. 1 above). But assemblies of billions of stars (galaxies) and trillions of stars (clusters) cause a lot of bending and are much more interesting.

Remember that the discovery that our universe has more than just one galaxy and later on a great many galaxies didn't happen that long ago. In the "Great Debate" at the Smithsonian in 1920, Harlow Shapley argued that the Universe was composed of only one big Galaxy; and in his model, our Sun was far from the center of this great island Universe. This view died after 1929 due to Hubble; and Andromeda was called a galaxy rather than a nebula after 1924. Fritz Zwicky became aware of the incredible mass of the Coma cluster in 1933 and wrote a paper "Nebulae as Gravitational lenses" in 1937. The **first** identified gravitationally lensed object was observed as two images of a single quasar– the famous "Twin Quasar " [QSO SBS 0957+561 A/B, **1979**, 6 arcsecond separation, mentioned on page 231 and shown on page 249]. Our old favorite text [Gravitation, MTW] came out before that in 1973 and barely mentions gravitational lensing.

NOTE: The word "lens" is usually misleading in the sense that unlike optical lenses, general case gravitational lenses lack a focal length (e.g., see top of page 309). Instead of a faithful image of a source, we can get arcs and rings.

Angular Momentum (page 218) is defined as **L = r × p** where momentum p = mv. Momentum (in the absence of external force) and angular momentum (in the absence of applied torque) are conserved. For a trajectory or orbit of a particle, mass doesn't vary – so lets ignore it. Angular velocity is $\omega = v/r = d\theta/dt$; so lets rewrite r ×v as $r^2 (v/r) = r^2\omega = r^2 d\theta/dt$ = a constant. The constant r×v = b×c or just bc, where the "impact parameter" b is the closest distance from the "undeflected path" to the bending mass M (Figure 7.6), e.g., the center of a galaxy or cluster of galaxies.

The famous **Hyperluminous galaxy** IRAS FSC 10214+4724 is mentioned on pages 217 and 219 and 206 {"InfraRed Astronomy Satellite" – "Faint Source Catalog"}. It has a redshift of z = 2.286 {and wavelengths scale as (z+1), so its Ly α peak in figure 5.8 pg 167 occurs at 121.6 nm x(2.286+1) = 400 nm}. It is a QSO that is gravitationally lensed and magnified by a factor of ten in infra-red.
We are also familiar with a 400 nm "Balmer Break" (recall hydrogen orbital transition n=7→2 gives 397 nm [and almost a continuum for higher n's], and high n ∞ → 2 is at 364 nm). This break was emphasized previously in figure 4.6 (pg 130). **Figure 7.5** shows a spectrum for this lensed galaxy with a break near 760 nm {which is 400 nm x (0.9+1=z+1) }. The spectrum is depressed below the break due to the presence of metals from old cold stars.

Figure 7.7 (page 220) shows two "alpha" angles and has skimpy wording. It is the upper "alpha-hat" angle that equals the Einstein deflection $\alpha^{\wedge} = \phi = 4GM/bc^2 = 4GM/\xi c^2$, where xi=ξ is the impact parameter. The observed source at S_1 is displaced from the actual source S by $\Delta S = S_1 - S$ to the right. All angles are small, so $\tan\alpha \sim \alpha \sim \Delta S/D_s$ and $\tan\alpha^{\wedge}$ (hat) $\sim \alpha^{\wedge} = \Delta S/D_{LS}$; so $\alpha = D_{LS}\alpha^{\wedge}/D_s$. $\theta \simeq \tan\theta = \xi/D_L$, $\eta = \beta D_s$, and angle $\beta = \theta - \alpha$, or $(\theta - \beta)D_s = D_s\,\alpha = D_{LS}\,\alpha^{\wedge}$ -- this leads directly to equations 7.12 and 7.13.

Magnification **$\mu = (\theta d\theta)/(\beta d\beta)$ Equation 7.16** (page 224) for a circularly symmetric lens is presented as if it should be obvious—but is it? Maybe yes if you're bright and fresh; but that wasn't me. The idea of lens magnification here requires that the source (e.g., galaxy) be an extended body extending beyond some central point S in the plane of the source (not a point but rather a solid angle). For a unit sphere in spherical polar coordinates, an element of solid angle is a little rectangle of area
$d\Omega = 1\, d\vartheta\, (\sin\vartheta d\varphi)$. Figure 7.7 is only two dimensional (say z up and x across and polar angle ϑ where $\vartheta = \beta$ for the source and $\vartheta = \theta$ for the image. Magnification is $\mu_x = d\theta/d\beta$, and the sideways magnification in a "y-direction" is $\mu_y = \sin\theta\, d\varphi\ /\sin\beta d\varphi$ for the same

dφ. So total magnification is the ratio between the solid angles of the image and the source = image area/source area =$\mu_x \mu_y$= [sin(θ)dθ]/[sin(β)dβ], but for small angles θ and β so that sine angle is similar to just little angle in radians.

Non-symmetrical lenses (top of page 221). In general, deflection angles are two-component vectors (α_1, α_2) in directions 1 perpendicular to 2 —a rectangular angular area. When lenses have axial (circular) symmetry, they can be effectively considered in terms of simple angles (like Fig. 7.7). However, galaxies and clusters may have complex density profiles or shapes resulting in an "elliptical" character (major and minor axes to a first approximation). Different lens models are discussed in section 7.7. More elaborately, μ = determinant [$\partial \theta_i / \partial \beta_j$] – note that pages 225 and 234 refer to an "inverse magnification tensor A with μ = 1/det[A= ($\partial \beta_i / \partial \theta_j$)]). For axial symmetry, A is diagonal; and det[A] = $A_{11} \times A_{22}$.

Between equation 7.24 and 7.25 is a term $(\sigma_v /220 \text{ km s}^{-1})^2$ which we saw before (e.g., page 195 and 202) for "M-σ " relations. Orbiting stars near a big black hole of mass near 10^8 suns have a dispersion near σ =200 km/second – it is just a convenient reference speed. The middle of page 239 says that 200 km/s is "typical of stars in the Galaxy." Quasars are associated with super-massive black holes.

"SIS" Singular isothermal sphere model, page 227: **The ideal gas law** is pV = NRT = NN_Ak_BT or p = ρ k_BT where N_A is Avagadro's number, N is number of moles of gas, k_B is Boltzmann's constant ~86 micro electro-volts/kelvin, and ρ = number of particles/volume is a number density. If m = mass of a gas particle, then ρ_m = ρ·m is a mass density, and p = $\rho_m k_B T/m$ (the equation mentioned in the middle of the page). The crucial assumption for the SIS model is the **$p \propto \rho_m$** starting point.

Now k_BT for one-dimensional motion is a tiny energy approximated by $m\sigma_v^2$ where v is the velocity of a gas particle. Saying that **m is the mass of a star** in a gas-like volume of stars doesn't really literally go with kT – unless k and T are strongly re-interpreted and merely motivate a "thermal" model. But using $m\sigma_v^2$ does make sense for kinetic energy and can be used for stars. It is an observable. Instead of little energy per gas particle, it is BIG kinetic energy per star; and rho is stars in a volume. And most of the mass in a cluster of stars is in the dark matter halo—which isn't overtly mentioned here but may obey similar equations with similar distributions to this model.

The solution for density ρ(r) in equation 7.21 is a fairly big jump from the previous equation. It might be better to approach the problem according to Poisson's equation for Newtonian gravity with potential Φ = -MG/r, $\nabla^2\Phi = 4\pi G\rho$ (for radius and spherical symmetry) instead: e.g., see http://www.iucaa.in/~dipankar/ph217/isothsph.pdf

The top of the page 230 says that **$d\theta /bc = dt/r^2$,** and this takes a little geometric thought. See figure 7.6 p 219 where the dashed line is r, and add a little extra dθ above the shown theta and a little cdt = dx to the left of the left vertex. The quantity rdθ corresponds to a short line down and perpendicular to line r on the left vertex. (rdθ)/(cdt) = cosθ = b/r , so it does work out.

The equation for **index n** on the top of page 231 has its signs reversed. n=c/v, so the top sign is minus and the bottom sign is +.

The Hubble parameter on the bottom of page 231 is H_o~ 72 (h = 0.72) which is in-between Planck 67.6 and more recent 74's.

Figure 7.24. Another more recent interesting picture is at: https://www.universetoday.com/142923/meet-our-neighbour-the-local-void-gaze-into-it-puny-humans/ There is also an amazing 3-D video.

Exercise 7.9 on page 233 (and text at top of page) refers to **"saddle points"** versus points of inflection – what's the difference? "An inflection point is a point on a curve at which the sign of the curvature (i.e., the concavity or second derivative) changes" such as y=sin x at x=0. A saddle point is a point of a function or surface which is a stationary point (meaning derivative = 0 = locally flat) but not an extremum. The function $y = x^3$ is both a saddle and an inflection point. For several variables the function $f(x, y) = y^2 - x^2$ has a saddle point at (0, 0) which is actually shaped like the middle of a saddle (like on a horse). The variables for time delay surfaces are θ_x and θ_y and "images form at stationary points." The author's labeling of merging points is a tad unclear.
Note that equation 7.46 (p 234) should read Tr[A] = a + b rather than ab.

Caustics: Wikipedia says that the term comes from Latin/Greek for burning – and concentrated light from the sun can cause burning. A caustic surface is a enhanced brightness geometrical envelope of nearly parallel optical rays. A common "cusp" shape is a curvy "V" shape like $y= |x|^{1/2}$ or Λ from (e.g.,) $y = - |x|^{1/2}$.

Equation 7.57 on page 242 is obviously in error {from Tr[A] on the previous page}. It should be $1 – ½ (\psi_{11} + \psi_{22})$.

Dark Energy Equation of State, DE EOS (**Figure 7.26)** is mentioned without elaboration here (but previous discussion was on page 56 and page 84). Page 56 says that the EOS is $p = w\rho c^2$ (so that w=0 means no pressure at all). Presently, w seems to be exactly w= -1 (negative pressure from Λ). But in case it varies with z, let $w(z)= w_o+w'_{z=0}\, z/(1+z)^p$ and plot w' versus w. Figure 2.18 suggests that the first derivative correction term is w' = 0., no contribution yet.

A recent astrophysics article [https://arxiv.org/pdf/2002.01479.pdf] used many of the concepts mentioned in this chapter: source and image planes, cluster halo masses (up to 300 trillion suns), singular isothermal sphere density profiles ($\rho \propto 1/r^2$), surface mass density Σ, critical value, magnifications above μ ~ 10, and Einstein radius θ_E. The article says that **gravitational waves** themselves can be magnified by gravitational lenses ! and can impact future LIGO statistics.

Now many multiple images of active galaxies have been seen (Einstein Cross).In this article, it is also mentioned that in 2015, a **supernova** was seen in four images (Kelly, ScienceMag).