BLACK HOLES

Addenda Notes for Serjeant Chapter Six

Dave 12/20/19-12/31/19 [update 1/1/20]

The Kerr ergosphere for a rotating black hole: Serjeant doesn't show any pictures, so take a look at the nice figures shown on pages 15 – 18 of: http://faculty.washington.edu/goussiou/486_W15/Soberi_BlackHole.pdf

For a free thousand page book on General Relativity, Black Holes, and Cosmology, Mike Jones has suggested an online text from a course by Andrew Hamilton (JILA - 2019).
Hamilton https://jila.colorado.edu/~ajsh/courses/astr5770_18/grbook.pdf
[Remember that the popular MTW 1300 page text GRAVITATION was from 1973 {and , very unfortunately, had set $\Lambda = 0$!}. This new text is updated, advanced, and uses a more modern formulation (differential geometry, forms, multi-vectors, tetrads, spinors, Penrose diagrams,…).

The Penrose Cosmic Censorship Hypothesis of 1979 (mentioned on page185) says two things: Weak form: an observer far away can never see a "naked" singularity because it will always be clothed with an event horizon. And Strong form: for ALL observers, general relativity (GR) is deterministic from initial conditions. This form was just proven to be false! That is, there are multiple solutions rather than one unique one – so GR fails for rotating BHs. E.g., see Quanta: https://www.quantamagazine.org/black-hole-singularities-are-as-inescapable-as-expected-20191202/

"Eddington Luminosity," L_E, is the light-power [watts] from a luminous source that can counter-balance the force of gravity on plasma-ions in the gravitational field of a massive source such as a star. The light pressure acts mainly on electrons (m_e, byThomson scattering). But plasma is neutral with equal numbers of electron charges versus proton charges. The mass of a unit volume of plasma is mainly the mass of its nuclei (often mainly protons), so gravitational force on ions is mainly on m_p's for the case of hydrogen plasma.

Accretion Disk: This chapter is about black holes so that the central gravitating mass here is labeled as M_{BH} ; and the source of luminosity becomes its orbiting accretion disk. Light originates from accelerating charges as friction-collisions and as a fraction, **(eta) η** , of infalling accretion matter converting its gravitational potential to energy {efficiency is < 1 because of loss of matter crossing the even horizon}. For non-rotating black holes, η ~ 6% (Eqn. 6.22) and for neutron stars it is about 10%. The loss of accretion disk matter results in an increase of black hole mass, so rate of loss could be expressed as either dM_{BH}/dt or $dM_{accretion}/dt$.

Due to friction, turbulence and magnetic fields, matter in an accretion disk will migrate inwards to the "last stable orbit" $r = 3R_s$ (e.g., bottom of page 192) and then should fall in. A sentence on the middle of page 194 says that, "not all the accreting matter falls into the black hole." A puzzle you may have is that since time freezes at the horizon, we won't ever see anything fall into the BH. But in its own free-falling frame of reference, light and matter fall right through the horizon without even noticing it. One other factor is that accreting matter that will fall in causes the horizon to increase outwards. The particles don't cross the horizon, the horizon crosses them {as a picture example, see the Figure in my book-2 on page 308 – the event horizon radius anticipates the future infall}.

{Errata: the paragraph following equation 6.8 should be corrected to an e-folding time of $\eta t_E/(1-\eta)$. Also, on equation 6.25, the two integral signs for dz and dS should be close together on the left side. The integrands are not separable.}

Exercise 6.1 {as you probably already know} Force in kg·m/s^2 is also expressed in newtons N (SI convention is small letters for newtons, big letter for unit N). For momentum, p = h/λ =hf/c = energy/speed of light. Equivalently, E = pc for light.

Section 6.3 presents what might be an initially abstract and challenging problem in the field of mathematics called "The Calculus of Variations" and ends up with a more directly useful "Euler-Lagrange equation" {"EL", equa**tion 6.16**}. {An easier and more tangible introduction is shown in my new book http://www.sackett.net/DP_Stroll2.pdf on pages 41-43, 47, and Noether theorem on pp 328-332}.

"The Principle of Least (or stationary) Action" is fundamental and applies to the fields of mechanics, optics, E&M fields, special and general relativity, other field theories, and particle physics.

ACTION A = ∫ Ldt where L is a "Lagrangian" function from time t_1 to t_2. A goal is to find pathways over which any variation in the action is zero:
$\delta A = 0 = \delta\int Ldt$, so $\delta\int 2Ldt = 2\delta\int Ldt = 0$-- which says that if L works, then 2L will also work. For ordinary mechanics, we choose **L = KE – PE.** When there are no changes in potential energy, we can also use A = ∫ pdx. For a small particle case, de Broglie p = h/λ , and action is then obviously proportional to a **"Wave Counter",** A = h(Δx/λ) ! – that is an interpretation of the meaning of action.

So, in Bohr orbits, for example, Δx = circumference of a circular orbit =C ; and orbital n'th wavelength is λ_n=C/n so that Action = hC/(C/n)= nh = Planck's cnst times the number of wavelengths in a circular electron orbit.

And what does the "EL" equation 6.15 mean intuitively?

dL/dx = (d/dt)dL/dv for L = KE(v) – PE(x) and KE = ½ mv^2 .

Now, dL/dv = ½ (2mv) =mv , and d(mv)/dt = mdv/dt = ma for the right side. And the left side is d(-PE)/dx which is force F. So **F = ma, Newton's law.**

The final portion of **section 6.3** for the rotating Kerr black hole is pretty advanced general relativity (e.g., page 911 of Misner, Thorne, Wheeler (MTW) reference) – meaning that we will just have to take his word for it.

The content of Section 6.4 very interesting! The total mass density of super-massive black holes in the universe can be inferred by optical counts of Quasars which provide a luminosity and energy density of QSO light. The density is about 0.01% of stellar density. Then expand this by about x4 to include type II active galaxies (e.g., p.207). Note that there are two Andrzej Soltans , father Polish nuclear physicist 1897-1959, and his Astrophysicist son b 1949.

Sphere of Influence of Black Hole, r_h, equation 6.29 Section 6.5 p 195.

Consider a particular mega-black-hole of mass 100 M suns and a central velocity dispersion near σ ~ 200 km/sec (as in the **"M-σ"** relations below), and also approximate an orbiting velocity as v ~ σ m/s. This is nearly the case for the barred spiral galaxy NGC

1097 (2015). Note that the term "stellar velocity dispersion" σ is also used for much broader cases than just the locality near central black holes. For example, recall that the Faber–Jackson relation applies to elliptical galaxies (e.g., $L \propto \sigma^4$).

CONSTANTS: $G=6.67\times10^{-11}$ m^2/s^2kg, $D_\odot$ =1 AU distance to our sun=1.5×10^{11}m. 1 Parsec = pc = 3.26 ℓy = 2.06×10^5 AU = 3.08×10^{16} m. $M_\odot = 1.99 \times 10^{30}$ kg.

The Schwarzschild radius $R_s \equiv 2M_{BH}G/c^2$, and it turns out that a 10^8 suns SMBH has an R_s near 2 AU (see eqn 6.30). Balance of forces on a star:
Centripetal force $CF=v^2/r \sim \sigma^2/r$ = gravitational force = GM_{BH}/r^2. So, $\mathbf{r_h = GM_{BH}/\sigma^2}$.
Plowing through all the conversions can give an equation of the form of eqn 6.29, **but** you need to multiply 6.29 in your text by x11 (it was in error—off an order of magnitude!). So, the considered values of M and sigma give an 11 parsec range of influence.

For perspective, the distance from us to α-centauri is about 4.37 ℓy = 1.3 pc. Our Milky Way BH Sgr A* has a mass of about 4.1 million suns and has a bunch of nearby stars in orbit about it.

{Exercise 6.6: I had an old note on extra wording needed at the end. **Add** :
"There is only time for 10.4 e-foldings in order to grow a BH large enough, it must be spinning more slowly and therefore have lower accretion efficiency.
"… and you still cannot create a supermassive BH by z = 2 through Eddington-limited BH growth if it accreting with maximal efficiency η = 0.42.}

The bottom of page 195 (our book 2010) says that, except for Quasars, the evidence for SMBH's in galaxies lacks direct detection. But now in 2019 we have the Event Horizon Telescope [EHT] direct imaging of the SMBH in galaxy M87 along with observations of stars and gas orbiting the center of our galaxie at very high speeds. The individual stars orbiting SgrA* make it clear that the Milky Way has a central SMBH. Very clear evidence for central SMBH's exists for M31 (Andromeda), M32, and 4395 and likely NGC 1277.
However, their mechanisms of formation are still unclear.

"Messier 87 (M87-- also known as Virgo A or NGC 4486) is a behemoth elliptical galaxy that sits some 53 million light-years from Earth. This giant holds trillions of stars" and has a very long jet stream from its core. Its SMBH has a mass near six billion suns!

"Reverberation mapping (p. 201) is an astrophysical technique for measuring the structure of the broad emission-line region around a supermassive black hole at the center of an active galaxy, and thus estimating the hole's mass" [WIK]. Details are discussed in { https://arxiv.org/pdf/astro-ph/0407538.pdf }.
Figure 6.9 needs elaboration: it shows a burst from the center onto orbiting clouds at radius r (time delay r/c) which emit their stimulated response towards the observer (another delay of x/c=rcosθ/c with respect to the central vertical dashed line). The total delay for an orbit point (r,θ) is then Δt = (r+x)/c as shown in the light dotted lines. Let the leftmost point of the circle be point P. The arc shown in the figure is a circle from point P of radius PY = P to point Y. There is then another point "-Y" of clouds at the lower (dotted lines vertex) that has the same time delay, Δt.

"M – σ" Relations: [Pages 202 {Figure 6.10}, 207-209]. Plots of the log of central black hole masses M versus the log of the central stellar velocity dispersion σ

gives pretty nice straight lines, $M_{BH} \propto \sigma^5$. A SMBH mass near 10^8 suns has a dispersion near σ =200 km/second. This strange tight linearity suggests that the dinky size black hole somehow affects the formation of galactic bulges. Perhaps there is momentum transfer feedback due to a wind that acts back on accretion flow. σ is easily measured and allows inference of unknown SMBH masses.

AGN Seyfert galaxies types 1 and type 2 are mentioned on page 207 (also see page 141). “As well as Seyferts, other galaxies are also classified as AGN. These include radio galaxies, quasars, blazars and LINERs.” “LINERs (“low-ionization nuclear emission regions”) were mentioned previously on page 142.

6.10 Merging Black Holes and Gravitational Waves (not yet seen by 2010). But we now have n ~28 BH+BH mergers beginning with GW150914 (September 14, 2015) see https://en.wikipedia.org/wiki/List_of_gravitational_wave_observations

These mergers show that about 5% of the binary mass is lost to gravitational radiation in forming the final remnant. That is, the energy of the gravitational waves can be more than a whole sun’s mass ($E= M_{\odot}c^2$).