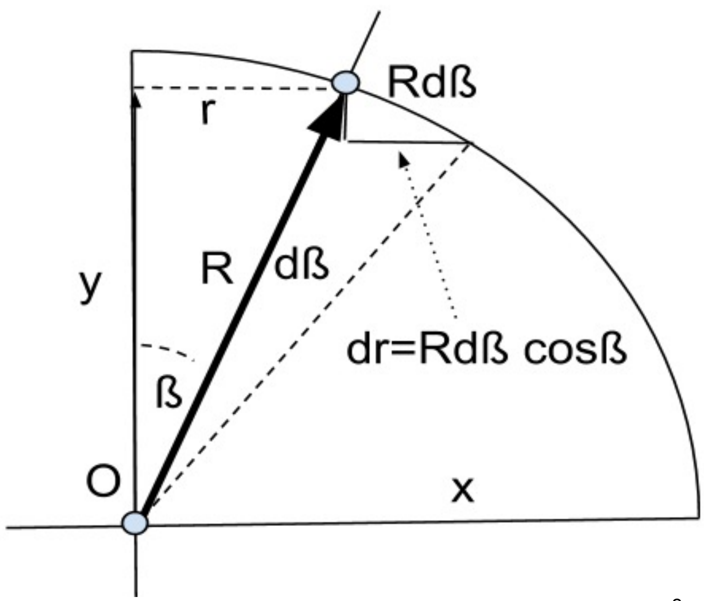
Addenda to Observational Cosmology Chapters 1 & 2

Dave 8/23/19 -9/11/19

For Meeting on 9/16/19+: Some comments and special additions that perhaps "should" have been somewhere in our new Book for Cosmology and might answer some questions. Chapter Two is longer and harder than Chapter One.

The Space Metric for a Basketball:

Serjeant just states a metric for a spherical space S^3 in eqn 1.6 (and uses it in 1.37). Where does his $\mathbf{dr^2/[1\text{-}kr^2]}$ term come from? It helps to first have a clear explanation for a simplest case like the surface of a basketball {or spherical shell, S^2 } with polar angle θ , longitude angle ϕ and radius R. That is easily done by examining a curve portion like that shown in the **Figure** below. Pick any longitude, say ϕ = 0, and only look at circular arcs in θ. Pick a point on the sphere and let r be the "radius" to that point from a y-axis.



The usual differential angle space metric here is $(d\ell)^2 = R^2(d\theta^2 + \sin^2\theta\, d\phi^2)$, so an element of θ arc {or β in the figure} has familiar length $d\ell = Rd\theta$, and the element of length around a latitude is $R\sin\theta d\phi = rd\phi$ where $r = R \sin\theta$. Examine a tiny differential triangle having acute angle θ again, hypotenuse Rdθ, altitude dy, base $dx=dr=Rd\theta \cos\theta$. Now $\cos \theta = y/R$ where $y = \sqrt{(R^2 - r^2)}$, so $Rd\theta = dr/\cos\theta$; and $\cos\theta = (1-\sin^2\theta)^{1/2} = \sqrt{(1- r^2/R^2)}$.

So, $(d\ell)^2 = (Rd\theta)^2 + (r\, d\phi)^2 = \mathbf{[dr^2/(1- r^2/R^2)]} + r^2 d\phi^2$.

And, the curvature of a sphere is "k"= $+1/R^2$. ……
…And then we play games with cosmological scale and scale factors and address "three-sphere" metrics embedded say in 4-dimensional Euclidean space. We could now discuss S^3 using three angles: θ, ϕ and a new "hyperpolar angle" chi, χ, that we can't

easily picture). Then, it will be the new $(Rd\chi)^2$ term that will be equal to $dr^2/[1- r^2/R^2]$ in equation 1.6.

A touch of History for expanding cosmology:

Einstein proposed his static universe cosmology in 1917 using λ as a term counteracting gravity (at that time, our milky way was the whole universe – so the idea of a homogeneous isotropic universe was inspired – or convenient). de Sitter immediately published his universe without matter using only λ. Then in 1922 Friedman considered a dynamic radius of curvature R = R(time) – his new universe could expand or even oscillate. In 1927, Lemaitre also proposed an expanding universe. Einstein rejected both proposals. In 1930, Eddington stated that Einstein's 1917 static world solution was unstable and might easily expand or contract. So, in 1931 Einstein finally agreed that the model of the universe should be a dynamic one like Friedman's and abandoned the cosmological constant.

{See "Einstein's conversion" at https://arxiv.org/pdf/1311.2763.pdf }.

Cosmological Distance in Chapter One:

{I have to relearn this every time; and the only way to feel comfortable with it is via a lot of playing around}.

Brief Summary: We seem to have **seven** (or more **) types of distance!**
One is just ruler or metric distance d_p between masses ("proper" distance separation at the same time – any time, not limited to light emission and absorption). Or, we could say, "Cosmological proper distance" between two points measured along a path defined at any constant cosmological time ($d_p = a(t)\ \Delta R$). In Chapter two, Serjeant uses $r_p = \int cdt/R(t)$ as proper distance. Eqn. 2.12.

Then three deduced light distances. Let "then" be a time when a galaxy emitted light and "now" when we receive it. Emit distance d_e is 'emit to receive' distance both at time = "then" = "d_p then." Look-back time or "light travel" distance $d_{LT} = c\Delta t$ from there and then to here and now. **Comoving** distance $d_C = \int cdt/a(t) = \int^z_o cdz/H(z)$ includes the expansion of space from "there and then" to "here and now" -- where the source and receiver are now, d_c is d_p "now" and so is also called d_{now} or d_o (i.e., when $a(t) = a_o = 1$).

Distances ordering is $\mathbf{d_{emit} < d_{LT} < d_{now}}$.

We also use $d_{hor} = d_{\ horizon} = \int cdz/H(z)$ from 0 to ∞ (from emit time ~ zero!). The "Particle" (or cosmological or comoving or light) Horizon is the maximum distance from which light could have traveled to the observer over the age of the universe – the size of the observable universe.

Three observed distances: Angular diameter distance d_A= object diameter/$\Delta\theta$; "proper motion distance" from transverse speed $d_M = v_\perp / \Delta\omega$ where $\omega = \Delta\theta/\Delta t$ -- also coincides with $r_e = r_o - r_{emit}$ or "coordinate distance measure." And there is Luminosity distance d_L using observed light flux.
$d_A = a^2 d_L$ and $d_M = ad_L$ (and $d_A = ad_M$, $a \leq 1$), so ordered distances are $\mathbf{d_A < d_M < d_L}$.

For more, see "Misconceptions" at https://arxiv.org/pdf/astro-ph/0310808.pdf and http://astro.pas.rochester.edu/~aquillen/ast142/Lecture/cosmo.pdf and

The **<u>Text Equation 1.33</u>** for Hubble ratio $H(z)/H_o = E(z)$ is <u>important</u>, is used, presents problems, and looks like it deviates from everything I've ever previously seen: {Such as Peebles' Cosmology pg. 100: $(H/H_o)^2 = \Omega_{mo}(1+z)^3 + \Omega_{ro}(1+z)^2 + \Omega_\Lambda \equiv$ "$E^2(z)$". Similarly, Misner,Thorne,Wheeler {**Gravitation**, "The telephone book"} eqn. 27.40 is nearly the same but with scale a instead of z.

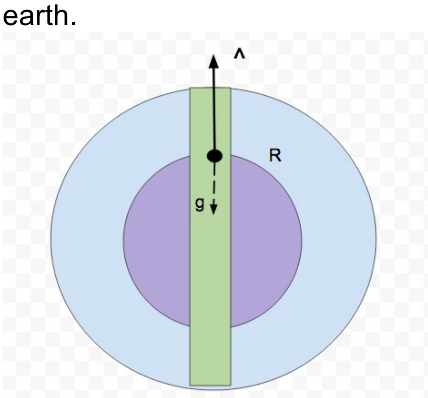
$(a\ \mathrm{dot})^2/a^2 = -k/a^2 + \Lambda/3 + (8\pi/3)(\rho_{mo}\, a_o^3/a^3 + \rho_{ro}\, a_o^4/a^4)$ } .

The H/Ho = E(z) formula by Serjeant must work ok but is hard to "grok." {He set $\Omega_r = 0$ here and discarded curvature k}. He uses his equation in 1.34 and again in 1.44 &1.56 . {Bill Daniel has written out the algebra for the derivation of 1.33.} Withou t radiation, Serjeant's equation has limited range {Chela has commented on this}-- perhaps out to $z \leq 5$ -- which is adequate for <u>Observational</u> cosmology.

<u>EdS</u> The **<u>"Einstein-de Sitter"</u>** cosmological model of <u>1932</u> has only mass $\Omega_{m,o}=1$ and $\Lambda = 0$ (even though the de Sitter universe was <u>all</u> Λ). It has the great virtue of easy calculations in closed form (vs numerical integration otherwise) and works fairly well for 300<z<2. So it is good for homework exercises (like Ex.1.4, 1.5, eqn 1.45, Eqn 4.7 and for simple understandings). It was very **popular** for <u>many</u> years—even in 1980 when it was discovered that $k \simeq 0$. Many books now don't even mention it {…I don't like to discard history}. In section 2.7, the "particle horizon" for an EdS universe is $d_{\mathrm{proper}\infty} = d_{hor} = 2c/H_o$.

<u>The simplest Way to introduce Cosmic Inflation: (see Section 2.7- 2.8)</u>

A thought problem for a cylindrical shaft filled with vacuum going all the way through the earth.



The accelerating expansion due to inflation can be related to the freshman physics problem of the motion of a ball falling through a long hole dug through the center of the earth. At the surface of the earth, the gravity is g_o (e.g., 9.8 m/s^2). At any other radius away from center, the mass of the earth that contributes to attraction is only the mass inside a spherical "Gaussian surface" at that radius, R. Near the center, that volume is tiny so that there is little force. As the body moves outwards, there is more and more attracting mass below the ball, so the restoring force increases and the body comes to a halt.

Force = F= $-kR$ = mass·acc = m· d^2R/dt^2. The period of oscillation is found to be tau =$\tau = \sqrt{3\pi/\rho G} \simeq$ 1.4 hours (where average earth density is 5.52 g/cc) . The ball simply falls through the earth to the other side and then back again. Because of the negative sign; the solution is just simple harmonic motion like that of a spring with a restoring force – a **SINE** Wave.

Now switch to **Λ** and change signs on the spring constant! **-** → + .
Inflation with a huge cosmological constant and with p = -ρ would end up with a net negative -2p anti-source causing effectively a **repulsive gravity** which makes the universe `fall outwards.' Or, we might consider a spherical shell of `pebbles falling outwards.' This form has a repulsive force **F = +kR,** a similar but different differential equation. Every step away from the center of the earth sees more "mass" behind it with more and more repulsive force. Instead of sine-wave motion, the solution this time is a runaway **exponential expansion**! {a "little" difference is that inflation has no "center."}
[exercise: plug $R = R_o \sin\omega t$ and also $R = ke^{+bt}$ into $d^2R/dt^2 = \pm kR$ to show that the signs work out right]. The inflation solution is:
$\mathbf{R(t) = ke^{\{+bt\}}}$ where $b = \sqrt{\ } \{8\pi G\ \rho\ /3\}$.
Two problems are, ``how does it start and how does it end?"

https://en.wikipedia.org/wiki/Inflation_(cosmology)
http://w.astro.berkeley.edu/~jcohn/inflation.html

The discussion of inflation in our book sections 2.7,2.8 is not easy to grasp with clarity.
Recall the two Friedman equations (1.7 &1.8): a first order one with a $(dR/dt)^2$ term and dynamic one of order two with a d^2R/dt^2 term. Given an intense scalar "inflaton" field with huge energy density V(ϕ), the dynamic equation produces an initial fast expansion that can be dampened by friction. Then, in the other equation, this expansion quickly makes any curvature contribution negligible ($k/a^2 \rightarrow 0$, p.56 eqn.1.7, 2.22,2.24) leaving a "possible" Λ and a residual scalar potential field V(ϕ) which can be considered nearly constant due to a "slow roll" nearly flat potential. I'll just lump these together into some new huge effective Λ (not our "traditional" or current cosmic constant Λ). A resulting $(dR/dt)^2 \sim \Lambda c^2R^2/3$ has a solution $R = R_o e^{\sqrt{\ } (\Lambda c2\ /3)t} = R_o e^{H\,t}$ {rapid **exponential** growth! – like the repulsive gravity above}.
Serjeant avoids most of this commonplace simplicity and just ends up saying $\mathbf{H^2 \propto V(\phi)}$ {eqn. 2.24, which amounts very roughly to the simple math above} with no further discussion -- as if you should know what it means! (This equation is similar to the old de Sitter equation on pg. 36).
Note that there are so many different versions of inflation theory that it might not be falsifiable (possibly meaning "beyond science").

Planck Mass, m_{Planck}, using h, c, and G : Natural Planck Units were suggested in 1899 **before** the Black Body radiation paper of 1900 that introduced what was later called Planck's constant, h (Mike and I are still not sure how). {Ref: M. Planck. Naturlische Masseinheiten. Der Koniglich Preussischen Akademie Der Wissenschaften, p. 479, 1899} (m_{Planck} is used in our book, Section 2.7). The fields ϕ used in Inflation are near this mass energy ! (see answers 2.7 p. 295).

The scale invariant power spectrum (p. 63) with equal energy per octave can also be called 1/ f noise or Pink noise, and has a "random fractal structure."

The Speed of Sound, c_s, in the Universe at the time of Recombination,

(CMB, z ~ 1000): … is a sizeable fraction of the speed of light!

For a photons-only (very early) universe without mass, $c_s=\sqrt{(p/\rho)} = \sqrt{(c^2/3)} = c/\sqrt{3}. \simeq 0.58c$. But after the $\Omega_m \sim \Omega_r$ equality near z ~ 24,000, the inertia of matter begins to alter and reduce this speed. "Acoustic Peaks" Page 73 says that the speed of sound relative to the speed of light is $\beta = c_s/c = (3 + 2.25\Omega_b/\Omega_r)^{-½}$, so we need to know the baryon to radiation ratio.

Eqn 1.15 is $\Omega_r=8\pi G\rho_r/3H^2$. Then $\Omega_b/\Omega_r = \rho_b/\rho_r = \rho_{bo}/a^3 \ / \ \rho_{ro}/a^4 = a(\Omega_{bo}/\Omega_{ro})$ now. At present, the Ω fractions for "matter" (in this case being "dark matter"), baryons and radiation with h ~ 0.7 is roughly
$(m_o, b_o, r_o) \sim (0.26, 0.043, \sim 2\times10^{-5})$ or $\Omega_{bo}/\Omega_{ro} \sim 2150$ – highly matter dominated!

Then z ~ 1000 says that temperature at recombination is near 3000K which drops to the present TBB ~ 3K. Then, (a~ 0.0009)x(2150) ~ 1.98, so β ~ 0.45c. Exercise 2.9 uses β ~ 0.58 – OK, but not exactly right. $c_s/c=1/\sqrt{3}$ is a commonplace conventional reference.

After the CMB, light pressure no longer counts and $c_s \rightarrow (4c^2\rho_r/9\rho_m)^{½}$. Temperatures of radiation and matter become nearly the same.

2.16 "The polarization of the CMB" "The detection of B-mode polarized clustering would be terribly exciting…" (p.80) .. and, an announcement of such a discovery was made in 2014. BUT: [Nature Jan 2015]: "A team of astronomers that last year reported evidence for gravitational waves from the early Universe has now withdrawn the claim. A joint analysis of data recorded by the team's **BICEP2 telescope at the South Pole** and by the European spacecraft Planck has revealed that the signal can be entirely attributed to dust in the Milky Way rather than having a more ancient, cosmic origin. (Our Serjeant book came out in 2010).