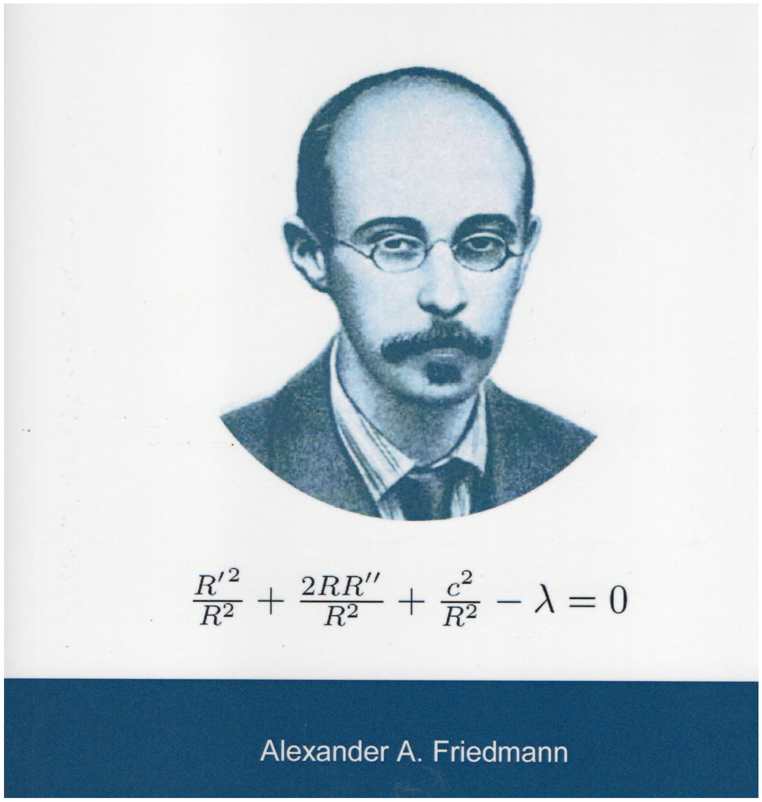
What follows is a note from Steve Crow purporting to do away with dark energy followed by a rebuttal from Bill Daniel. All part of the Boulder Cosmology Group's ongoing discussion of Serjeant's Observational Cosmology.

Comments on Page 19 of *Observational Cosmology*

Steven Crow, 16 August 2019



Stephen Serjeant introduces the Friedmann equations on page 19 of *Observational Cosmology* [1]:

$$\dot{R}^2 = 8\pi G\rho R^2/3 - kc^2 + \Lambda c^2 R^2/3\,, \qquad (1.7)$$

$$\ddot{R} = -4\pi GR\left(\rho + 3p/c^2\right)/3 + \Lambda c^2 R/3\,, \qquad (1.8)$$

where I have replaced the sum of matter and radiation density with ρ. Note that (1.7) and (1.8) both differ from the Friedmann equation on the cover of Friedmann's monograph *Papers on Curved Spaces and Cosmology* [2]. Some of the verbiage around the equations is muddled, but they are correct if used properly.

In particular, The Friedmann equations *do not* determine k and $R(t)$. The curvature k originates in the Robertson-Walker metric prior to the application of the Einstein field equations. Instead the Friedmann equations determine Λ and $R(t)$, where Λ is Einstein's cosmological constant. From (1.7),

$$\Lambda = 3\left(\dot{R}^2 + kc^2\right)/c^2 R^2 - 8\pi G\rho/c^2\,. \qquad (1.7)^*$$

Eq (1.8) becomes

$$\ddot{R} = \left(\dot{R}^2 + kc^2\right)/R - 8\pi GR\left(\rho + 3p/c^2\right). \tag{1.8*}$$

Eq (1.8)* is an ordinary second-order nonlinear differential equation for $R(t)$ containing the curvature k but not the cosmological constant Λ or its pseudo-physical correlate dark energy.

The first term on the right of (1.8)* provides the acceleration in the current epoch of the expanding universe. In the second term on the right, p can be expressed as an equation of state $p(\rho)$, and ρ can be expressed in terms of R by solving an equation for conservation of energy. For our universe, $k = 0$, and (1.8)* can be solved easily with current conditions

$R(t_0) = R_0$ and $\dot{R}(t_0) = H_0 R_0$.

The solution fits the latest distance-redshift data better than the ΛCDM model The age of the universe proves to be 13.94 Gyr, with a curent Hubble constant of 74.0 km/sec/Mpc.

Unortunately, the muddle in the paragraph around Eqs (1.7) and (1.8) in *Observational Cosmology* makes its way into subsequent portions of the text, especially in Section 1.7, The Flatness Problem. There is no flatness problem. The universe is flat $(k = 0)$ and always has been. Moreover, the curvature k has no functional connection with the density ρ. The muddle surrounding flatness and dark energy is not Stephen Serjeant's fault, but resuts from an almost religious zeal with which cosmologists embrace mathematical errors and the resulting paradoxes.

References

1. Serjeant, Stephen *Observational Cosmology*. Cambridge University Press, 2010.

2. Friedmann, Alexander A. *Papers on Curved Spaces and Cosmology*. Minkowski Institute Press, 2014.

Rebuttal to Steve Crow's Note for Page 19.

Let's begin using Steve's approach, and solve each of (1.7) and (1.8) for $\frac{\Lambda c^2 R^2}{3}$. We then eliminate the Λ term by setting these expressions equal to each other, resulting in,

$$\dot{R}^2 - \frac{8\pi G\rho}{3} R^2 + kc^2 = \ddot{R}R + \frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) R^2 \, .$$

[Note that this *does not imply* $\Lambda = 0$. We have simply removed Λ from this relation by setting two expressions for it equal to each other.]

If, as observational results show, the universe is flat, then $k = 0$ and the above can be rearranged (remembering that $H = \dot{R}/R$) to give,

$$\frac{\ddot{R}}{R} - H^2 + 4\pi G\left(\rho + \frac{p}{c^2}\right) = 0.$$

This relation applies at all times. *<u>At the present time</u>*, it becomes,

$$\frac{\ddot{R}_0}{R_0} - {H_0}^2 + 4\pi G\left(\rho_0 + \frac{p_0}{c^2}\right) = 0.$$

In a flat universe, the current energy density, ρ_0, must be the critical density, $\rho_0 = \rho_c$. The critical density, ρ_c, is a function of H given by,

$$\rho_c = \frac{3H^2}{8\pi G}$$ (equation (6.4) from Liddle, 2nd ed., p. 47), and thus,

$$\frac{\ddot{R}_0}{R_0} + 4\pi G\left(\frac{\rho_c}{3} + \frac{p_0}{c^2}\right) = 0.$$

Since the universal expansion is observed to be accelerating, $\ddot{R}_0 > 0$. The scale factor, R_0, and the critical density, ρ_c, are also positive. Steve is proposing a universe with, "no appreciable pressure," so, $p_0 \approx 0$. (Steve correctly assumes this for a universe composed of nonrelativistic matter with a very small contribution from radiation.) But, other than the p_0 term which is essentially zero, all terms on the LHS of the equation above are positive, so they can't sum to zero as required. This contradiction implies that the universe must also contain a component that supplies a negative pressure to offset the positive contribution from nonrelativistic matter and radiation.

That component is what cosmologists have labeled "dark energy" and most see it as arising from a cosmological constant, Λ. (Others propose a variable cosmological "constant" called *quintessence*, but if this turns out to be true, it wouldn't change this argument.) It develops a *negative* pressure proportional to its density, $p_\Lambda = -\rho_\Lambda c^2$. This allows the LHS of the above equation to sum to zero.

Steve's error is the *simultaneous* assumption that $\Lambda = k = 0$. One or the other can be zero, but not *both*. Since observations show $k = 0.000 \pm 0.005$, we can safely conclude that $\Lambda > 0$ and our universe does indeed contain dark energy.

Here is a later addition to the rebuttal from Bill Daniel that is relevant to the issue:

To see why Dave says that, while "there are <u>three</u> 'Friedmann' equations... only two of the three are independent," I had to start with <u>4 equations</u> in 5 variables (R, ρ, k, p, and Λ, and some of their time derivatives). The equations and my names for them are:

The "Friedmann equation":	$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2 - kc^2 + \frac{c^2}{3}\Lambda R^2$	(Serjeant 1.7)
The "Acceleration equation":	$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3}{c^2}p\right)R + \frac{c^2}{3}\Lambda R$	(Serjeant 1.8)
The "Fluid equation":	$\dot{\rho} + 3\frac{\dot{R}}{R}\left(\rho + \frac{1}{c^2}p\right) = 0$	(Liddle 3.15)
The "Work equation":	$\frac{d}{dt}(\rho c^2 R^3) = -p\frac{dR^3}{dt}$	(Serjeant Ex 1.3)

In Exercise 1.3, Serjeant derives the Acceleration equation from the Friedmann equation *plus the Work equation*. Thus, this set really represents, at most, three independent equations. What I missed on my first pass through all of this is that the Work equation from Serjeant and the Fluid equation from Liddle are identical (do the derivatives and rearrange to see this). So actually, as Dave notes, there are only ***2 independent equations*** in 5 variables. While we could take any 2 of the first 3 as independent, for simplicity sake, it makes a lot of sense to use the Friedmann and Fluid equations.

What Steve has done is to solve one of them (the Friedmann equation) for one of the variables (Λ) and plug it into another equation. It's because he plugs into the <u>Acceleration</u> equation (with its "hidden" dependence on the Fluid equation) that made his approach so confusing. This eliminates one of the equations (it doesn't matter which, but let's say it is the Acceleration equation) and this is why Dave says "the Friedmann equation F_1 should still also accompany it."

To this point, what Steve has done is completely legal (though confusing). It's exactly the approach we would take to solve any system of two equations: solve one of the equations for one variable and plug it back into the other equation. Recall carefully your *Algebra II* class (far back in the mists of time for some of us!): this operation eliminates one variable (Λ in Steve's case) *and one equation* (here, the Fluid equation – remember, the Acceleration equation is now really the Fluid equation in disguise). Note carefully the italicized phrase. It will become important later.

As Dave and I have both pointed out, Steve goes wrong at this point by assuming that this requires $\Lambda = 0$. It does not. To see why this is a mistake, suppose our two independent equations were *y = x + 1* and *y = 2x* (the variable *y* in this case is equivalent to the expression including Λ in Steve's analysis). Since the LHS of both equations is *y*, we can set the RHSs equal: *x + 1 = 2x*, and solve to find *x = 1*. (This last is the simple equivalent to Steve's equation (1.8)*.)

Note that plugging x = 1 into either of the original equations, we find y = 2 ***not*** y = 0. Steve does not take this further step, and I don't blame him. In the case of the cosmological equations, it's much more complicated (and not very enlightening) to solve for another of the variables, plug back in, and solve for Λ, but this simple example illustrates the mistake Steve is making by taking $\Lambda = 0$.

Instead, Steve correctly notes that his (1.8)* is a second order ODE in R. He also correctly eliminates the term in k (data from the Plank satellite indicates $k \approx 0$).

But (and here's the punch line), he then states that "p can be expressed as an equation of state $p(\rho)$, and ρ can be expressed in terms of R by solving an equation for conservation of energy." That "equation for conservation of energy" is just the Fluid equation above. But remember, *we already eliminated the Fluid equation* (masquerading as the Acceleration equation) when we created equation (1.8)*. *It's not mathematically justified to resurrect this equation and pretend that it is useful for the elimination of another variable* (p). In fact, it's just this mistake that leads him to the erroneous conclusion that $p = 0$ and hence $\Lambda = 0$ that I pointed out in my previous "rebuttal."

I must say that the remainder of his paper and the figure are a complete mystery to me, but this analysis is sufficient to convince me beyond any doubt that the errors in his paper are so grievous that I don't want to spend any more time on it.