Mass States, Flavor States, Mixing, and Neutrino Oscillation

A Note to p. 214-5 of *The Mystery of the Missing Antimatter*, by Helen Quinn and Yossi Nir

Jeff Grove, May 2014

Descriptions of quark mixing and neutrino oscillation have been very confusing to me They all seem to describe the two phenomena independently, while I expected to find much in common between them. Our very well written current book, *The Mystery of the Missing Antimatter*, by Helen Quinn and Yossi Nir, comes closer to clarifying this issue, but still does not address it explicitly. On about the fourth reading of p. 214-5, I saw what I believe is the source of confusion. Quinn and Nir discuss the issue more closely than most authors, and a close reading of those pages, followed by a lot of digging on the Web, led me to write this note. My central idea is that the confusion results from an unmentioned difference in the way quarks and leptons are described in nearly all of the literature.

I will begin with a quick examination of some passages from the book. Then I give my take on how to explain this situation. I include a very detailed description of some basic material to establish common terminology, since I believe that disconnects in terminology are at the root of the confusion. Finally, I give a summary of a section of a Web page that addresses this issue more completely than I have seen elsewhere.

On page 76, Quinn and Nir introduce the idea of mass and flavor states as alternative descriptions of particle properties, although without using exactly those phrases. Page 135 introduces universality as a requirement of gauge symmetry in the standard model. The idea (there described for the strong interaction) is that the strength or probability of a given interaction class must be the same regardless of the flavor of the specific particle involved. Page 138 discusses universality of the weak interaction strength, and mentions that it works as expected for leptons, but fails for quarks. Pages 151-3 discuss this difference in a way I did not find very helpful, except that it emphasizes the difference between the mass and flavor states. The last paragraph of the section explains that universality is restored for quarks when probabilities for all quark (mass) states are compared with the probability of a single lepton (flavor) state, although it doesn't say why.

Pages 214-5 contain the bulk of this topic in the book. The statement (p. 214, ¾ down), "For quarks we found that the universal strength of the weak interaction was shared or mixed among quarks of different mass…when a given up-type quark absorbs a W-boson." shows the situation from the quark description perspective. A few lines later, "Each charged lepton converts to a single type of neutrino when it emits or absorbs a W-boson" appears to be a completely analogous statement, but of opposite meaning, from the lepton description perspective. Why should the weak interaction cause mixing in the quark types but not the lepton types? Quark states change under weak interaction, but lepton states don't? The authors come close to spelling it out, but don't actually say it. **<u>It turns out that quarks are normally described by their mass states, and leptons by their flavor states.</u>** Since the flavor states are the eigenstates of the weak interaction, they are not mixed in these interactions. But the mass states are superpositions of flavor states, so when a mass state (such as a quark in standard terminology) undergoes a weak interaction, the quantum incompatibility of the two kinds of states forces the result to be statistically selected from the three possible flavor states. That's mixing.

If the last paragraph makes sense, you don't need to read the rest of this note. What follows is a very picky description of basic concepts, worded to make the usage of terminology as clear as possible. I had to read several other sources (see "Further Reading" below), and write the rest before I could write the last paragraph.

Fermions and Their Mass and Flavor States

Fermions, the particles that make up matter, come in 12 varieties, or "flavors". There are six quarks (ignoring the three colors of each, and all antiparticles) and six leptons. The six quarks are associated into three doublets (generations), with common names up/down, charm/strange, and top/bottom (u/d, c/s, and t/b). The upper members of each doublet, u, c, and t, are called the "up-type" quarks, and the others are the "down-type". The six leptons are also in three doublets, called neutrinos (the upper doublet members, ν_e, ν_μ, ν_τ), and electrons or "charged leptons" (the lower doublet members, e, μ, τ). These commonly used designations are sufficient for most of what is usually said about these particles.

However, when it is necessary to talk about the generation mixing that occurs in conjunction with the weak interaction, this categorization becomes confusing. It does not describe the forms of the particles that lead to the mixing. There is another way to describe each of these particle flavors, and it is between these alternative descriptions that the mixing occurs.

The alternative descriptions are called the "mass states" and the "flavor states" of the particles. The flavor states could also be called interaction states, since these states determine what happens when a particle interacts with another. These two sets of states become more than just descriptions because, quantum mechanically, as with position and momentum, they cannot both be known simultaneously. If you detect one, you force a change in the other. So if a particle is known to have been created or detected by a particular interaction, its flavor state is known with certainty. That requires that its mass must be a superposition of the mass states that are available to it (the mass *eigenstates* – defined below), rather than a specific one of them. Likewise, if a particle is subjected to an operation that determines a specific mass state, its flavor state must then be a superposition of the possible flavor eigenstates.

The actual relationship of the particles and their descriptions in terms of these two kinds of states is the same for quarks and leptons. When a neutrino of a given flavor interacts, it always involves a charged lepton (electron) of the same generation. No mixing is apparent. Yet when a quark interacts, the other quark in the interaction is a superposition of quark states in all three generations (i.e. mixing has occurred), in contradiction of the first sentence of this paragraph. How is this possible?

This apparent contradiction arises because the terminology used differs between quarks and leptons. It is rarely mentioned unless this specific issue is being discussed. **In the usual terminology, leptons are known by their flavor states, but quarks are known by their mass states.**

Since neutrinos can be detected only by interactions, it is natural to use the flavor states to describe them. Then, when we talk about weak interactions of leptons, we are already talking about their flavor (interaction) states, and the interaction proceeds without generation mixing. It is only when some process occurs that involves neutrino mass that the mass states matter. One such process is free propagation through space. In that case, the velocity or wavelength of a neutrino matters, and this follows the quantum rules of the mass states. After propagation for some distance, we still can only detect the neutrino by its interactions. But the interaction states have been mixed by the mass-dependent propagation process, so neutrinos of known flavor have become mixed flavor. This is what neutrino oscillation means.

The situation is reversed for quarks. Since they are identified by their mass states, any particle interaction leads to forced mixing of mass states. But when these two conventions are combined, the verbal gymnastics necessary to make the overall situation clear are often omitted. For example, on p. 214-5, it is stated that the weak interaction strength (probability) found between an electron and a neutrino of a single generation is shared among the three generations of quarks. This is called "a different pattern". Actually, it isn't. Since interactions are determined by interaction or flavor state, the interaction probability will necessarily be shared across all three generations when mass states are considered, regardless of whether quarks or leptons are involved. This is a consequence of the fact that for the weak interaction, the interaction or flavor states are not exactly the same as the mass states. Each group of states is a superposition of the states of the other group. For unknown reasons, the alignment of the two sets of states is fairly close (i.e. not too much mixing) for quarks (CKM matrix), but much more mixing occurs between leptons (PMNS matrix).

Eigenstates

The set of particle states that may result when a particular operation (or measurement) is performed on a particle is the set of eigenstates of the operation. If an operation is performed on a particle, and it was already in one of the eigenstates of the operation, it will go into the specific state that the operation requires (which will also be one of the eigenstates), with probability 1. If it was not in one of those eigenstates, then it was in a superposition of them (because any state can be defined that way). The result will still be one of the eigenstates, but which one will be probabilistic. The probabilities will depend on the relative amounts of the different components of the operation's eigenstates in the superposition. Performing the operation forces the particle into one of the operation's eigenstates, after which it is unequivocally in the particular eigenstate that was selected probabilistically.

My interpretation of http://danielscully.co.uk/thesis/neutrinos.html#section-mixing:

In the case of the weak interaction, the fermion states under particle interaction ("flavor states", which are what determine particle interactions) are not the same as their states for other purposes ("mass states"; not sure just what operations these are associated with). (For electromagnetic and strong interactions, the interaction states are the same as the mass states.) Each of the 12 fermions can be described both as superpositions of the three mass states of its type-mates and of their flavor states. If a particle is in a particular eigenstate of either set, the superposition is (1,0,0) in terms of that set of eigenstates, but can be thought of as (x,y,z), where $x^2+y^2+z^2 = 1$, in terms of the other set. So far, everything is completely symmetrical with regard to the kind of states used to describe the fermions, and upper vs. lower members of each weak doublet.

We could arbitrarily choose to use one of the state types (flavor or mass) as our standard to describe the composition of the other state types. We could also arbitrarily choose the upper or lower members of each group of three doublets to be the reference states, and the opposite triad to be the set of states in which the mixing (superposition) occurs. (I don't understand this separation.) In fact, there are conventions for these descriptions, and they are not the same for quarks and leptons. That difference contributes to the confusion in discussing mixing.

To get more specific, the usual quark states (u/d, c/s, t/b) are the mass states, and the upper triad (u,c,t) are used as the reference, so all the mixing is deemed to occur in the lower triad (d,s,b mass states, or d',s',b' flavor states). The usual lepton states (ν_e/e, ν_μ/μ, ν_τ/τ) are the flavor states, and the electron types in the lower triad are the reference, so all the mixing is deemed to be in the neutrinos (ν_e, ν_μ, ν_τ, flavor, or ν_1, ν_2, ν_3, mass). (In the case of leptons, these choices make sense, since neutrinos are very hard to observe, and can presently be identified only by their interactions (flavor). In fact the only operation I am aware of that relates to the mass states of neutrinos is letting them propagate and observing that the resulting superposition of flavor states changes (neutrino oscillation).)

We had a discussion in a meeting about the "reality" of different kinds of states. Scully states:

> One will sometimes find one or other of the flavour/mass states being described as the "physical" states (usually mass in quarks and flavour in neutrinos) but such a description is misleading. It is the measurement being made which determines which states are important, and any perception that one set is more "real" than another is simply an artefact of the view the experimenter has.

Further Reading

See also http://profmattstrassler.com/articles-and-posts/particle-physics-basics/neutrinos/neutrino-types-and-neutrino-oscillations/. The first nine paragraphs (before neutrino oscillations) have a similar explanation of states and mixing.

For more depth, see also:
http://en.wikipedia.org/wiki/Cabibbo%E2%80%93Kobayashi%E2%80%93Maskawa_matrix#The_matrix
http://en.wikipedia.org/wiki/Cabibbo%E2%80%93Kobayashi%E2%80%93Maskawa_matrix#Weak_universality
http://en.wikipedia.org/wiki/PMNS_matrix#The_matrix