CHAPTER 11

11.1 THE FACTOR OF 3 IN Ω_ν COMES FROM THE NEUTRINO FAMILIES AND THE FACTOR OF $\sqrt[3]{4/11}$ BECOMES $\left(\frac{4}{11}\right)^{4/3}$ BECAUSE OF THE 4TH POWER TEMPERATURE DEPENDENCE IN EQN (11.12). CALCULATING THE FACTORS IN EQN (11.1) GIVES

$$\frac{\Omega_\nu}{\Omega_{rad}} = 3 \times \frac{7}{8} \times \left(\frac{4}{11}\right)^{4/3} = \boxed{0.68}$$

11.2 SINCE THE ENERGY OF THE NEUTRINO BACKGROUND, ε_ν, IS ABOUT THE SAME AS THAT OF THE CMB, ε_γ, AND FROM PROBLEM 11.1 THE ENERGY PER NEUTRINO IS $E_\nu = \left(\frac{4}{11}\right)^{1/3} E_\gamma$, THEN BY EQN. (5.24),

$$n_\nu = \frac{\varepsilon_\nu}{E_\nu} = \frac{\varepsilon_\gamma}{\left(\frac{4}{11}\right)^{1/3} E_\gamma} = n_\gamma E_\gamma \Big/ \left(\frac{4}{11}\right)^{1/3} E_\gamma \text{, so}$$

$\boxed{n_\nu = 1.4\, n_\gamma}$ AND THE NEUTRINO FLUX IS $\Phi_\nu = n_\nu c$.

IF WE GUESS THAT THE SURFACE AREA OF THE HUMAN BODY IS ABOUT $1\,m^2$, THEN

$$\Phi_\nu A_{BODY} = n_\nu c A_{BODY} = (1.4)\, n_\gamma c A_{BODY}$$
$$= (1.4)(3.7\times10^8\, m^{-3})(3\times10^8\, m s^{-1})(1\, m^2)$$
$$\boxed{= 1.6\times10^{16}\, sec^{-1}}$$

11.3 $T_\odot = 10^7 K$ USING EQN. (11.12)

$$\left(\frac{1s}{t}\right)^{1/2} = \frac{10^7 K}{1.3\times10^{10} K} = 7.7\times10^{-4}$$

$$t = 1/(7.7\times10^{-4})^2 = \boxed{1.7\times10^6 s \cong 20\,days}$$

11.3 Cont. I DO NOT GET LIDDLE'S ANSWER OF 4×10^{6} s.

THE UNIVERSE AT THIS TIME WAS RADIATION DOMINATED

$E_{CERN} = 10^{11}$ eV

AGAIN, USE (11.12)

$$\left(\frac{1\,s}{t}\right)^{1/2} = \frac{E_{CERN}}{1.1\,MeV} = \frac{10^{11}\,eV}{1.1\times10^{6}\,eV} = 9\times10^{4}$$

$$t = 1/8\times10^{9} = \boxed{1.2\times10^{-10}\,s}$$

AGAIN, I DO NOT GET HIS ANSWER OF 4×10^{-10} s.

$$T = E_{CERN}/k_B = 10^{11}\,eV/8.6\times10^{-5}\,eV\,K^{-1} = \boxed{1.2\times10^{15}\,K}$$

11.4 IF WE ASSUME THAT $\Omega_{rad}/\Omega_{\nu}$ IS THE SAME AS IT IS TODAY, THEN EQN (11.1) LEADS TO

$$\Omega_{rad} = \Omega_{rel} - \Omega_{\nu} = 0.32\,\Omega_{rel}.$$

EQN (11.4) HOLDS AT DECOUPLING AND GIVES

$$\Omega_{rad}(t_{dec}) \cong \frac{(0.32)(0.04)\,\Omega_{mat}(t_{dec})}{\Omega_0 h^2}.$$

AN INTERMEDIATE STEP IN THE SOLUTION TO PROB. 7.5 WAS:

$$\Omega_{mat}(t) = \frac{\rho_0/a^3}{\rho_0/a^3 + \rho_0\left(\frac{1}{\Omega_0}-1\right)}.$$

SINCE $a_{dec} = 0.01$, $\Omega_{mat}(t_{dec}) \approx 1$, AS WE WOULD EXPECT, AND TAKING $\Omega_0 \cong 0.3\,h^{1/2}$ AND $h = 0.72$

$$\Omega_{rad}(t_{dec}) = \frac{0.32}{0.30}\cdot\frac{(0.04)}{(0.72)^{3/2}} = \boxed{0.07}$$

A BIT BIGGER THAN LIDDLE'S 0.04.

CHAPTER 12

12.1 ALL NEUTRONS WOULD HAVE DECAYED INTO PROTONS:

$$\frac{N_n}{N_p} \simeq \frac{1}{5} \exp\left(-\frac{(340\,\text{sec})(\ln 2)}{10^{-6}\,\text{sec}}\right) \cong \varnothing$$

SO NO ATOMS BEYOND HYDROGEN WOULD FORM AND THE UNIVERSE WOULD HAVE BEEN DEPRIVED OF THE PLEASURE OF MY PRESENCE.

12.2 THE NEUTRON/PROTON NUMBER DENSITY AT TIME t_{nuc} AFTER THAT WHEN $k_B T \simeq 0.8\,\text{MeV}$ (THAT IS, 340 sec.) IS:

$$\frac{N_n}{N_p} \simeq \frac{1}{5} \exp\left(-\frac{340 \cdot \ln 2}{100}\right) \simeq 0.019 \cong 1/53$$

SO THE MASS FRACTION OF ^4He IS

$$Y_4 = \frac{2}{1 + N_p/N_n} = \frac{2}{1 + 53} = \boxed{0.037}$$

12.3 THE # BARYONS $= N_B = N_n + N_p = \left(1 + \frac{1}{7.3}\right) N_p = 1.13\, N_p$

UNIVERSAL CHARGE NEUTRALITY REQUIRES $N_p = N_{e^-}$,

SO $\frac{N_{e^-}}{N_B} = \frac{1}{1.13} \frac{N_{e^-}}{N_p} = \boxed{88\% \simeq \frac{8}{9}}$

CHAPTER 13

13.1 SINCE $|\Omega_{TOT} - 1| \propto t^{2/3}$ IN A MATTER-DOMINATED UNIVERSE, $\Omega_{TOT} \to \infty$ AS $t \to \infty$. THIS MEANS $\rho_c \to 0$ AND THE UNIVERSE BECOMES EMPTY.

13.2 SEE ANSWER IN BOOK

13.3 THE EQUATION QUOTED AS (11.12) IS INCORRECT. IT SHOULD READ: $\left(\frac{1\,\text{sec}}{t}\right)^{1/2} \simeq \frac{T}{1.3\times10^{10}\,\text{K}}$

USING $T = 3\times10^{25}$ K IN THIS EQUATION RESULTS IN

$$t = \left(\frac{1.3\times10^{10}}{3\times10^{25}}\right)^2 \text{sec} = \boxed{1.9\times10^{-31}\ \text{sec}}$$

(USING THE INCORRECT EQUATION RESULTS IN THE BOOK'S ANSWER OF 4×10^{-31} sec.)

FOR CURVATURE DOMINATION, $T \propto t^{-1}$, SO, USING THE ABOVE: $\frac{3\times10^{25}\,\text{K}}{3\,\text{K}} = \frac{t}{1.9\times10^{-31}\,\text{sec}}$, OR

$t = 1.9\times10^{-6}$ sec

(AGAIN, USING THE INCORRECT EQUATION RESULTS IN LIDDLE'S ANSWER OF 4×10^{-6} sec)

13.4 IF THE AGE OF THE UNIVERSE IS TAKEN TO BE $1/H = 1/10^{-10}\,yr^{-1} = 10^{10}\,yr$, AND LIGHT SPEED IS 3×10^{-7} Mpc yr^{-1}, THEN THE LIGHT TRAVEL DISTANCE IS $(3\times10^{-7}\,Mpc\,yr^{-1})(10^{10}\,yr) = 3000\,Mpc$

IF $a(t) \propto t^{2/3}$ THEN $\dot{a}(t) \propto t^{-1/3}$ AND $H = \frac{\dot{a}}{a} = t^{-1}$. SO, AT DECOUPLING, $H = (3.5\times10^{5}\,yr)^{-1}$ AND $d_{dec} = (3.5\times10^{5}\,yr)(3\times10^{-7}\,Mpc\,yr^{-1}) = 0.1\,Mpc$

SINCE, AT DECOUPLING, $\alpha \approx 10^{-3}$, THIS DISTANCE IS STRETCHED TO $(0.1\,Mpc)(10^{3}) = 100\,Mpc$.

THE ANGLE IS $\theta = \arcsin(100\,Mpc/3000\,Mpc) = 2°$. WITHOUT INFLATION, ANY LOCATIONS ON THE CMB SEPARATED BY $>2°$ COULD NOT HAVE BEEN IN THERMAL EQUILIBRIUM, SINCE THEY ARE CAUSALLY DISCONNECTED.

13.5 SINCE $T(t) \propto 1/a(t)$ (10.7) AND $\frac{\Omega_{MON}}{\Omega_{RAD}} \propto a(t)$ (11.3), THEN $T(t)\left(\frac{\Omega_{MON}(t)}{\Omega_{RAD}(t)}\right) = $ CONSTANT. AT $T = 3\times10^{28}$,

$a(t_{GUT})$ IS VERY SMALL AND SO IS t_{GUT}. AT THE GUT

TIME $\Omega_{RAD} \approx 1$, SO THE CONSTANT ABOVE IS

$$\text{CONSTANT} = T(t_{GUT})\,\Omega_{MON}(t_{GUT}) = 3\times10^{28}\times10^{-10}$$
$$= 3\times10^{18}$$

SO, WHEN $\frac{\Omega_{MON}}{\Omega_{RAD}} = 1$, $T_{EQ} = 3\times10^{18}$ K.

NOW $T(t_0) \cong 3$ K, SO $\frac{\Omega_{MON}(t_0)}{\Omega_{RAD}(t_0)} = \frac{3\times10^{18}}{T(t_0)} = 10^{18}$

SINCE $\Omega_{RAD}(t_0) \cong 4\times10^{-5} h^{-2}$ (11.2), $\Omega_{MON}(t_0) \cong 4\times10^{13} h^{2}$,

ENORMOUSLY GREATER ρ_{MON} THAN ρ_{CRIT}. THERE SHOULD BE LOTS OF MONOPOLES!

13.6 THE INFLATIONARY EXPANSION MUST DRIVE Ω_{vac} UP BY A FACTOR OF 10^{18} FOR $\Omega_{MON}/\Omega_{RAD} = 1$

SINCE $\rho_{MON} \propto a^{-3}$ AND ρ_{RAD} = CONSTANT, THE SCALE FACTOR, a, MUST GO UP BY A FACTOR OF $\sqrt[3]{10^{18}} = 10^{6}$

A3.1 WITH $\hbar = c = 1$, $P = E = K_B T$, SO $\sigma = G_F^2 K_B^2 T^2$

$$\Gamma = J\sigma = nu\sigma = (K_B^3 T^3)(c)(G_F^2 K_B^2 T^2) = G_F K_B^5 T^5$$

$$\frac{\Gamma}{H} = \frac{G_F K_B^5 T^5}{\left(\frac{K_B^4 T^4}{10^{19}\,\text{GeV}}\right)} = (K_B^3 T^3)(1.2 \times 10^{-5}\,\text{GeV})(10^{19}\,\text{GeV})$$

$$= \left(\frac{K_B T}{6.9\,\text{MeV}}\right)^3 \approx \left(\frac{K_B T}{1\,\text{MeV}}\right)^3$$

A3.2 THE PARTICLES COMPOSING THE "SEA" ARE:

PARTICLE	FERMION/BOSON	FACTOR	DOF	TOTAL
e^-	F	7/8	2	7/4
e^+	F	7/8	2	7/4
γ	B	1	2	2

P.138 (2nd ED) DISCUSSES THE $e^+ + e^- \leftrightarrow 2\gamma$ PROCESS. AFTER $T < T_{CRITICAL}$, 2γ CANNOT CREATE $e^+ + e^-$ PAIRS AND ONLY $e^+ + e^- \rightarrow 2\gamma$ OCCURS. THE FINAL RESULT IS THAT THE "SEA" CONTAINS ONLY PHOTONS. SO INITIALLY THE D.O.F. FACTOR IS

$$\begin{array}{ccccccc} e^+ & + & e^- & + & \gamma & & \\ 7/4 & + & 7/4 & + & 2 & = & 11/2 \end{array}$$

AFTERWARD, WITH ONLY PHOTONS, IT IS:

$$\begin{array}{c} \gamma \\ 2 = 2 \end{array}$$

SO, IF $s = \alpha g_* T^3$ IS CONSTANT (WITH $\alpha = \frac{2\pi^2}{45}$)

A3.2 CONT

$$\frac{S_{BEFORE}}{S_{AFTER}} = 1 = \frac{\alpha\left(\frac{11}{2}T_{\nu}^{3}\right)}{\alpha\left(2T_{\gamma}^{3}\right)} \Rightarrow$$

$$T_{\gamma} = \sqrt[3]{\frac{11}{4}}\,T_{\nu}$$

A3.3 RAN OUT OF TIME