A1.1 a) THE FIRST TERM IN THE METRIC HAS POLES AT $r = \pm 1/\sqrt{K}$. SINCE r MUST BE POSITIVE, $\boxed{0 \leq r < 1/\sqrt{K}}$. NOTE: I DISAGREE WITH THE SOLUTION IN THE BOOK THAT ALLOWS $r = 1/\sqrt{K}$, SINCE THAT IS A POLE.

SUBSTITUTING $r = K^{-1/2} \sin(K^{1/2}\xi)$ AND $dr = \cos(K^{1/2}\xi)\,d\xi$ GIVES:

$$ds_3^2 = a^2(t)\left[\frac{\cos^2(K^{1/2}\xi)\,d\xi^2}{1-\sin^2(K^{1/2}\xi)} + \frac{\sin^2(K^{1/2}\xi)}{K}\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

USING THE IDENTITY $\sin^2\theta + \cos^2\theta = 1$,

$$ds_3^2 = \frac{a^2(t)}{K}\left[K\,d\xi^2 + \sin^2(K^{1/2}\xi)(d\theta + \sin^2\theta\, d\phi^2)\right]$$

THE METRIC IN HYPERSPHERICAL COORDINATES TAKES THE FORM:

$$ds^2 = r^2\left(d\psi^2 + \sin^2\psi\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right)$$

THIS IS IDENTICAL TO THE EXPRESSION FOR ds_3^2 ABOVE WITH:

$$r = \frac{a(t)}{\sqrt{K}}\,;\quad d\psi = \sqrt{K}\,d\xi \quad \text{AND} \quad \psi = \sqrt{K}\,\xi.$$

SO, THIS TRANSFORMATION RESULTS IN HYPERSPHERICAL COORDINATES ON THE 3-SPHERE.

A1.(b) FOR $K<0$, $\sqrt{K} = i\sqrt{-K}$

SINCE $\sinh(i\theta) = i\sin\theta$, TRY

$$r = (-K)^{-1/2}\sinh\left((-K)^{1/2}\xi\right) \rightarrow dr = \cosh\left((-K)^{1/2}\xi\right)d\xi$$

AND

$$ds_3^2 = a^2(t)\left[\frac{\cosh^2((-K)^{1/2}\xi)\,d\xi^2}{1-(-(-K))\left(\frac{1}{-K}\right)\sinh^2((-K)^{1/2}\xi)} + \frac{\sinh^2((-K)^{1/2}\xi)}{-K}(d\theta+\sin^2\theta\, d\varphi^2)\right]$$

USING THE IDENTITY $\cosh^2\theta - \sinh^2\theta = 1$,

$$ds_3^2 = \frac{a^2(t)}{K}\left[K\,d\xi^2 + \sinh^2((-K)^{1/2}\xi)(d\theta+\sin^2\theta\, d\varphi^2)\right]$$

SIMILARLY, THIS GIVES A TRANSFORMATION TO HYPERBOLIC COORDINATES ON THE 3-HYPERBOLIC PARABOLOID.

USING THE EXPRESSION FOR r ABOVE,

$$\frac{\text{CIRCUMFERENCE}}{\text{RADIUS}} = \frac{2\pi(-K)^{-1/2}\sinh\left((-K)^{1/2}\cdot 10(-K)^{-1/2}\right)}{10(-K)^{-1/2}}$$

$$= \frac{2\pi}{10}\sinh(10) = 6920$$

VERSUS 2π IN FLAT SPACE.

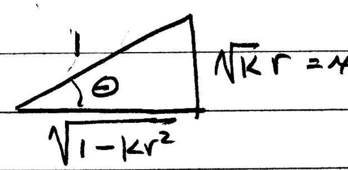
A1.2 SINCE WE ARE INTERESTED IN PHYSICAL (RADIAL) SEPARATION, $d\theta = d\phi = 0$. AT CURRENT TIME,

$$ds^2 = a_0^2 \left(\frac{dr^2}{1-kr^2} \right)$$

INTEGRATING TO GET THE SEPARATION, $(0 \le r < 1/\sqrt{k})$

$$s = \int \sqrt{ds^2} = a_0 \int_0^{1/\sqrt{k}} \frac{dr}{\sqrt{1-kr^2}}$$

THIS IS A CLASSIC TRIG SUB INTEGRAL WITH:



$$x = \sqrt{k}\, r = \sin\theta$$

$$dr = dx/\sqrt{k} = \cos\theta \, d\theta/\sqrt{k}$$

$$\sqrt{1-kr^2} = \cos\theta$$

$$s = a_0 \int_0^{1/\sqrt{k}} \frac{dr}{\sqrt{1-kr^2}} = \frac{a_0}{\sqrt{k}} \int_0^{\pi/2} \frac{\cos\theta \, d\theta}{\cos\theta} = \boxed{\frac{\pi a_0}{2\sqrt{k}}}$$