DERIVATIONS OF EQUATIONS
ADVANCED TOPIC 2

(A2.10) COMBINE (A2.9) WITH (2.1).

(A2.17) FOR SMALL θ, THE TAYLOR SERIES FOR $\sin\theta$ IS

$$\sin\theta = \theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots$$, SO

DROPPING HIGHER ORDER TERMS $\sin\theta \cong \theta$

(A2.18) (A2.1) WITH $r = r_0$ AND $dr = d\varphi = dt = 0$

(A2.19) BY (A2.10)

(A2.20) BY (A2.15)

(A2.21) FROM (A1.4) THE DIFFERENTIAL RADIAL ELEMENT IS $dr/\sqrt{1-kr^2}$. THIS DIFFERS FROM THE USUAL DIFFERENTIAL RADIAL ELEMENT IN SPHERICAL COORDINATES IN A FLAT GEOMETRY (JUST dr) BECAUSE OF CURVATURE. IN FLAT GEOMETRY THE VOLUME ELEMENT IS GIVEN BY

$$d\tau = r^2 \sin\theta \, dr \, d\theta \, d\varphi .$$

SO INSERTING THE NEW RADIAL DIFFERENTIAL AND MULTIPLYING BY THE SCALE FACTOR (BECAUSE THE GEOMETRY IS EXPANDING) GIVES (A2.21).

(A2.22) $dN = n(t)\,dV \,/\, \sin\theta\, d\theta\, d\varphi$. THE SECOND EQUALITY IS FOR PRESENT TIME.

(A2.23) NOT NECESSARY TO EVALUATE THIS INTEGRAL BUT NOTE THAT IT IS ANOTHER "TRIG SUB", USING $\sqrt{K}\, r = \sin \chi$.