Internal Space

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A quote:

"Prior to isospin, the symmetries of physics, such as translation invariance, rotation invariance, and Lorentz invariance, were confined to the spacetime we live and love in. Heisenberg's profound insight *{in 1932}* led to the discovery of a vast internal space, the ongoing exploration of which has been a central theme of fundamental physics for close to a hundred years now. Isospin has a wealth of falsifiable implications. For instance, Yukawa's pion field *{1947}* was expected to transform like a 3-component vector under rotation in this internal space" *[Anthony Zee, "Simply," pg. 197]* .

Terms in the literature: internal quantum numbers, internal symmetry, internal gauge symmetry, internal space, internal space operator, internal "charge" (e.g., "color"), internal rotation, internal direction, internal lines (propagators for "interaction fields"). The "Standard Model" of particle physics is an internal gauge theory with gauge group SU(3)×SU(2)×U(1). A big question is:

¿ *How "real" are the internal gauge symmetry groups and their "phase angles?"*

Below our familiar classical reality in space-time lies the unseen hidden arena of the quantum-amplitude world {"quantum-land"}. As a foundation for modern particle physics, we now also have quantum "Gauge Theories" based on "gauge symmetry groups." The word "gauge" is historical and misleading and usually just means quantum "phase" symmetries. These refer to the possible symmetry transformation of the "wave-functions" {or "spinors, ψ" } in the Lagrangians, $\mathcal{L}$, of the relevant "force" fields.

The strange "gauge game" is to imagine making local phase change, θ(x,t), to wave functions and then compensate for that by also changing potential fields and derivatives so that the triplet of changes are invisible to us in our experimental space-time world {we might call it four-way if we include the Lagrangian which then uses the modified derivative: "$\partial_\mu \rightarrow D_\mu$" that incorporates new "gauge" potential fields {like "A"}. Changing the quantum phase locally is already "strange" because it means changing the phase of a "matter wave" by action of a possibly fictitious electromagnetic "sub-potential," χ , a new scalar potential that can alter an existing scalar or vector potentials. A key equation is **$\theta(x,t) = q\chi(x,t)/\hbar$.**

"Imparting a local gauge symmetry to a field theory immediately leads to the appearance of a gauge field that can couple to the source of the conserved charge." Gauge bosons are the quanta of gauge fields and are considered as generators of the symmetry.

The triplet of changes reveal the forms of interactions for quarks and leptons. A goal is to **maintain the invariance of the Lagrangian** -- keep all the choreographed changes hidden. This "hiding conspiracy" makes discussions of "reality" difficult but yields remarkable results. During the last sixty years or so, physicists realized that all fundamental interactions were consistent with constraints imposed by using local gauge symmetries. In quantum electrodynamics {QED} for example, gauge symmetry pertains to both "electron matter waves" and electromagnetic {EM} waves and interrelates them. Later-on, it also revealed the world of "color" in quantum chromodynamics {QCD} and the vector bosons of the electro-weak interactions {the W's and Z 'heavy photon' particles associated with the combined "Lie group" $G = U(1)_y \times SU(2)_{left}$ broken to $U(1)_{EM}$ }. 1971 was a key year in which "Yang-Mills" {YM} theory was shown to be renormalizable even after the realization of the importance of "spontaneous symmetry breaking" {the "Higgs" mechanism}. This meant that it was now possible to deal with the infinities of quantum field theory {QFT} calculations in a systematic way. The leading current theories are all of the YM type.

Heisenberg's 1932 proposal of "isospin" was a foundational concept first for nuclear physics and later for particle physics in general. If electromagnetism is ignored, the neutron and proton look a lot alike with respect to strong interactions. Isospin postulated that n and p could be somehow "rotated" into each other in dominant strong interactions in analogy to electron spin up-versus-down states in a "spinor" [Heisenberg]. He was also motivated by the case of electron exchange binding in the hydrogen molecule ion H_2^+. The symmetry group of quantum spin is the continuous {special unitary} "Lie group" "SU(2)" {for more on Lie groups, see [**Renaud**], [Wikipedia] or [Kunasz] }. It is important to note that this isospin concept has nothing to do with usual quantum-mechanical "spin;" it just follows a similar mathematics. Roger Penrose defines a spinor as an object which turns into its negative after a complete $2\pi = 360^o$ rotation: $S(\theta + 2\pi) = -S(\theta)$ – acting in a sense like the square root of a vector. In 1954, **Yang and Mills** built on this isospin view as an SU(2) Lie group that could change its values <u>locally</u> rather than just globally {meaning everywhere at once} – "local gauge symmetry."

As electron spin is represented by a column spinor such as $(^+_-)$, there could be a nucleon {iso-} spinor $\psi = (^p_n)$ to be operated on by elements of the group – an <u>internal</u> <u>symmetry</u> transformation. The proton in terms of three quarks is written by the "ket" vector |uud⟩ while the neutron is |udd⟩, so we may now say the transformation reduces to that between u and d {"up" and "down"} quarks; or $\psi = (^u_d)$ with isotopic spin projection $I_z = +\frac{1}{2}$ and $-\frac{1}{2}$ {Heisenberg himself stated $J_z = +1$ for n and -1 for p}. "The weak decay of the down quark generates the decay of the neutron" [Zee Simple]. Turning all u quarks into d quarks and vice versa, u↔ d, can be accomplished by multiplying its isospinor by one of the quaternions matrices, $M = i\sigma_y$, where σ_y is a 2x2 "Pauli matrix:" $M\psi = (^{0}_{-1}\ ^{1}_{0})(^u_d) \rightarrow (^d_u)$. "± iσy" is also the generator of rotations for the continuous rotation group SO(2) ≃ U(1).

The basic <u>generators</u> of infinitesimal "little" su(2) {"the tangent space of the Lie group SU(2) at the identity"} are Hamilton's i, j, k <u>quaternions of 1843</u>– the first <u>hypercomplex</u> number system (with a basis of three different imaginary numbers}. But, it is a frequent convention to instead use the more standardized Pauli matrices $\{\sigma_x, \sigma_y, \sigma_z\}$ for operations on electron state spinors. *Technically, Pauli matrices are not really proper generators unless multiplied by "i" – which again makes them quaternions.* We rise to the full SU(2) group by "**exponentiation**" of these entities; and that includes $e^o=1$ which provides the group identity element. The 2×2 real matrix for rotations in the x-y plane $\{SO(2)=R_2\}$ is given by e to the matrix M times angle θ : $R_2 = e^{\theta M} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ {shown by expanding $e^{\theta M}$ as its definition in a power series and consolidating terms}. And in reverse, matrix M of so(2) is returned by taking the derivative of R_2 at the identity ($\theta \rightarrow 0$): $M = d(R_2)/d\theta|_{\theta=0}$. For su(2), we have a slightly more complex 2×2 matrix form, $\begin{bmatrix} ib & c+id \\ -c+id & -ib \end{bmatrix}$ for real variables b, c, and d ∈ ℝ . The more elementary case of the one-dimensional unitary group of circular rotations, U(1), has just a 1×1 matrix single number generating element that could be "i" or "1" since the circle group is like the real axis, ℝ , winding around a hoop.

The special transformations that Yang and Mills [YM] considered in 1954 were like $\psi' = \exp[-i\vec{\theta}\cdot\vec{\sigma}]\psi$ where **three** continuous phase angle theta's can now vary locally in space time $\theta_i = \theta_i(x,t)$. {Many books write this as $\psi \rightarrow \exp[(\frac{1}{2})(\tau \cdot \alpha)]\psi$ where tau's are Pauli matrices }. The "symmetry" of $\mathcal{L}$ is in the local symmetry of the possible phases in the wavefunctions ψ. Yang and Mills were hoping that local SU(2) transformations would eventually be useful for nuclear strong interactions. That turned out not to be true, but their general approach would be key to progress in particle physics. SU(2) was partly relevant to the weak interactions (like beta decay involving neutrinos), but a larger SU(3) group was needed for progress towards strong interactions and finally resulted in the quantum chromodynamics [QCD] of quarks and gluons.

These 3 θ's for the case of SU(2) increase in number to **eight** phase angles for the bigger group SU(3) {e.g., the "eightfold way" for mesons using u, d & s quarks}. Also note that the 8 3×3 complex "Gell-Mann" matrices, "T_a" for $SU(3)_{color}$ with its 8 generators for the case of quark "colors" {"charges" q= R,B,G} explains why there are 8 colored gluons acting between nucleon quarks. The transformation form again looks like $q(x) \rightarrow \exp[i\alpha_a(x)T_a]q(x)$ where there are now a=1 through 8 SU(3) angles α_a and 8 transformation matrices T_a. For tiny angles, an infinitesimal SU(2) transformation is just $\psi' \simeq (1+ i\theta\cdot\sigma)\psi$, and an isospin operator is $\hat{\mathbf{I}} = \int\psi^\dagger \vec{\boldsymbol{\sigma}}\, \psi d^3x$ [Ait p. 260] .

What we might consider as an initial

List of "internal quantum numbers" and "internal spaces" could include:

{Strong} **Isospin**, **I and I_z** connecting up and down quarks (and hence proton to neutron nucleons). Electric charge is $Q = I_z + ½ Y$ for "strong hypercharge, Y " {Special cases $Y_u=Y_d = 1/3$, and $Y_n=Y_p = +1$}. As discrete global symmetries, they correspond to useful conserved physical quantities. Internal gauge symmetries lie below any observable physical quantities.
Flavor symmetry: between all quark flavors {ignoring mass differences}. Strong isospin is a low-mass flavor subset. There are six flavors of quarks and six flavors of leptons.

{Strong} **Hypercharge**, **Y,** assisting isospin, electric charge and flavors u, d, and s. Y = B + S = baryon number + "strangeness" (strange quark) {but then, we later need also "charmness," bottomness (or beauty), and topness quantum numbers (once called truth) added on}.
"**Color**" Isospin and Color Hypercharge: I_3^c vs Y^c SU(3) triangles for R,G,B and anticolor $\overline{R}$,$\overline{G}$,$\overline{B}$.
$Y^c_r=Y^c_g = 1/3$, $Y^c_b = -2/3$, $I^c_{3r} =+ ½ = -I^c_{3g}$, $I^c_{3b} = 0$. Change signs for anticolor.
Weak Isospin I_w *or T_3* gauge symmetry of the weak interactions. . The weak isospin is the same as the electric charge because $Y_w = 0$. I_{w3} of $W^+ = +1$.
Weak hypercharge in the electroweak interactions, Y_w . Charge $Q = T_{3w} + ½ Y_w$.

These are all associated with discrete quantum numbers of importance to high energy particle physics. Isospin is conserved in strong interactions but can be violated for electromagnetic and weak interactions. The "s" quark, for example, decays slowly via a weak interaction {$s\rightarrow u+W^-$, $W^-\rightarrow e^-+\overline{\nu}_e$} . They may also have associated **continuous** gauge groups which are the special unitary groups SU(n)'s. "Each theory of fundamental interactions has two symmetry groups, a space-time group and a local group, the latter is intimately connected with interaction dynamics" [Auyang].

Some think of all of these as being only abstract or fictitious and physically unreal with the frequent definition of "real" often meaning "classically real" to us in our space-time. But if Nature actually uses something *isomorphic* to these concepts in internal spaces, then they have aspects of reality even if hidden to us and beneath classical space-time. There are ongoing debates about the "reality" of "inner spaces." These concepts are so extremely useful that we might assume some sort of reality "isomorphic to" their uses.

Continuing beyond these, note that every elementary particle has its own labeled universal quantum field extending throughout all of space-time. Particles are the excitations of the corresponding fields, and most fields are hidden until stimulated (or "fluctuated"). The phrase "internal fields" might apply for all of these.

It is not usually stated, but relevant complex and hypercomplex spaces might also be considered as "internal spaces" not easily seen in our classical world of mainly real variables. Even for ordinary quantum mechanics, we have wave phase as $\phi = kx - \omega t + \phi_o$ with particular reference phase ϕ_o not ever detectable and hence "meaningless" to us (labeled as a "**redundancy**"—and gauge theories are all about redundancies). The presence of complex waves themselves, $\psi \propto e^{i\phi(x,t)}$, are also not directly seen but may be deduced after experiment. So, even these simple complex waves live in a world below the classical world. And, just beyond this, electron spin is described using hypercomplex quaternions as basis generators of the Lie algebra su(2).

I am tempted to say that the inner quantum world is a "square root of reality" which is generally covered by physically relevant "Clifford algebras" [algebras encompassing "square roots" of -1 like the quaternions and also +1 such as that of the Pauli matrix algebra]. But, unfortunately, "unlike SU(2) which is isomorphic to Spin(3) and which therefore can be described via the Clifford algebra Cℓ(0,3) {for the number of "√ +1's" and "√ -1's" }, <u>SU(3) does not arise naturally in any Clifford algebra.</u> But, it is a group of fundamental importance in elementary particle physics - it is central to the Standard Model" [Renaud]. Also, many believe that the hypercomplex "octonians" with seven different imaginary number bases are useful in particle physics. But they are a non-associative algebra and hence <u>not</u> a Clifford algebra.

We might also distinguish between elements in a "space" such as spinors versus operators on the elements such as the SU(n) group elements. For Dirac theory, spinors exist at points in space, and operators include Dirac gamma matrices. The operators and elements of inner spaces go together.

The real interest in this note is the degree of "<u>reality" of the internal gauge symmetry groups</u> and their phase angles. Much about them is "**<u>unmeasurable</u>**" and hence in opposition to the usual pragmatism, positivism, and instrumentalism of Copenhagen quantum mechanics. But, "during the last six decades, Yang-Mills theory has become the **<u>cornerstone</u>** of theoretical physics—<u>the great "relevance of local irrelevance."</u> It is seemingly the only fully consistent relativistic quantum field theory in four space-time dimensions" [Shifman]. It provides a basis for the standard model by "providing a unified framework to describe the quantum-mechanical behavior of electromagnetism, the weak force and the strong force" and their experimental predictions" [wik.intro]. Some view gauge theory as just as significant as relativity and quantum mechanics. The renormalizability of QED also depends on gauge symmetry. Such high importance must imply some essential reality – but it is hard to pin down.

<u>QUANTUM GAUGE THEORY:</u>

First, a little necessary background review on **classical gauge theory** for electromagnetism. E and B physical force vector fields are real because they produce measurable experimental results and have energy densities proportional to E^2 or B^2 (and we might say that energy is the king of concepts in physics). Their nature is specified by "Maxwell's Equations;" and they are each allowed to be expressed using potential fields because of the vector identities **div curl ≡ 0 and curl grad ≡ 0.** That is:

Maxwell's "No magnetic Poles" equation, $\nabla\cdot\mathbf{B} = 0$, allows $B = \nabla\times A$ since $\nabla\cdot(\nabla\times A) = 0$. A is "vector Potential." {For a long time, people believed that these potentials were unreal and avoided them}.

There is "gauge freedom" in expressing A since $B = \nabla\times A = \nabla\times(A+\nabla\chi(x,t))$ with $\nabla\times\nabla\chi \equiv 0$. "Chi" can be any irrelevant but differentiable scalar function on space-time and has no effect on physical B.

Faraday's Law $\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t = -\partial(\nabla\times A)/\partial t = \nabla\times(-\partial A/\partial t)$ and also $= \nabla\times(-\nabla\phi)$ where $\phi(x,t)$ is the electric potential scalar function. This allows the electric field E to be expressed in terms of two potentials as $E = -\nabla\phi - \partial A/\partial t$. The addition of the ghostly unreal chi field must not affect E.

So, $E = E' = -\nabla\phi - \partial A/\partial t = -\nabla\phi' - \partial(A+\nabla\chi)/\partial t = -\nabla\phi' - \partial A/\partial t - \nabla(\partial\chi/\partial t)$, or $\phi' = \phi - \partial\chi/\partial t$.

Combining these together in relativistic 4-vector notation, we have $\mathbf{A'_\mu \rightarrow A_\mu + \partial_\mu\chi}$ {the modern notation ∂_μ means $\partial/\partial x^\mu$ with index $\mu = 0,1,2$, and 3 where zero means "time-variable" t}. The χ field is scalar with single values in space-time and has values $\Delta\chi = \int\chi\cdot d\ell$ independent of chosen path.

The **"gauge function" χ(x,t)** is initially seen as an unusual little beast – a strange "potential for calculating another potential." We've likely never encountered anything like this before.

"Choosing gauge" or gauge conditions such as the familiar "Coulomb" gauge or "Lorenz gauge" become useful when we are trying to solve for potential fields beginning with wave equations for potentials that involve terms like $\nabla^2\phi$ and $\nabla^2\vec{A}$ (a vector Laplacian). Selecting gauge conditions crosses-out terms in the wave form and enables easier calculations for $\phi(x,t)$ and $A(x,t)$. In the classical case, rather than "phase" transformation, gauge transformation just means an exploitation of the redundancy in potentials. Phases become important in quantum theory.

Regions of space where there is no magnetic field, B=0, should have $\vec{A} = -\nabla\chi(x,t)$ – a simple gradient field. An important exception is the case of the "Aharonov-Bohm [AB] effect" for electron wave paths around a long-thin solenoid. These solenoids may have a strong interior field $B = B_o > 0$ along with extremely weak exterior magnetic fields. But, they still have a significant exterior $\vec{A} > 0$ field that can wind around the solenoid's center "hole" and are "dragged" around by rotational currents. $\vec{A}_{out}(\rho) = \vec{A}_\phi(\rho) = \mathbf{B_o}R^2\hat{\phi}/2\rho$, where R is the radius of the solenoid, ρ is a radial distance and $\hat{\phi}$ is a unit vector in the "phi-angle" direction. This winding field is multi-valued and not a true gradient field because pathways are not simply connected due to the hole. The AB effect is not changed by the introduction of a $\nabla\chi(x,t)$ field.

In this example, the quantum mechanical AB $\vec{A}_\phi$ field has some well-tested quantum **reality** because it causes an observable electron interference phase shift between parts of the wave function:

$\psi_{AB} = e^{i\Delta\theta} = \exp[(ie/\hbar)\int A\cdot d\ell\,]\ \psi_{A=0}$ which is path dependent (and we consider a full circle path about a solenoid). This leads to a view of the vector potential field that Maxwell entertained—"eA" is like an electromagnetic momentum as if an electromagnetic space were being dragged along with current flows. And a moving-frame Lorentz transformation with relative velocity v of a Coulomb electrostatic field produces a vector $\vec{A}$ field. That also dovetails with canonical momentum being $p_c = p_k + eA$ where $p_{kinetic} = mv$. The quantum phases here are said to be "kickable" – we kick the system by increasing solenoid current, and it responds by visibly shifting electron phase interference patterns. Being kickable is a measure of physical reality [Auyang].

Gauge Theory in Quantum Field Theory:

"The term 'gauge' refers to any specific mathematical formalism to regulate redundant degrees of freedom" [Wik]. The point of local quantum "gauge" symmetry {*usually now meaning quantum "phase" symmetry}* is that "it constrains the form of the action, S, and dictates the form of the interaction," $\mathcal{L}_{int}$ [dp2013]. "The gauge principle provides a method to transform a Lagrangian, $\mathcal{L}$, which is invariant under a global symmetry into a Lagrangian that is invariant under a local symmetry." "For physics to be invariant under a group of transformations, it is only necessary for the action to be invariant [Zee]." For the case of ordinary quantum mechanics, we change the phase of a wave locally: $\psi \to e^{i\theta(x)}\psi$ where local phase shift angle $\theta = q\chi(x)/\hbar$ {where it is almost an axiom that electric charge, q, is absolutely conserved}. The "gauge parameter" factor $u=e^{i\theta(x)}$ is an element of the phase group U(1) of complex rotations. To actually achieve phase-shifting requires the existence and action of a potential on a wave associated with a moving charge. Physicists see electromagnetic E and B physical fields that can result from a potential A-field expressed in a variety of ways due gauge freedoms that do not affect the physical fields (redundancy and under-determination of potentials).

We play a game of compensating for the local phase shifting by tweaking the potential field A(x,t) "connection" due to charges and currents and also by modifying space-time derivatives to "covariant" derivatives, D, that incorporate the effects of the potential fields. For a given gauge symmetry group, this derivative is defined uniquely; and all ∂_μ's in $\mathcal{L}$ can be replaced by covariant derivatives. Then, "this uniquely specifies the interaction between matter and the gauge fields."

This is a careful and creative 3-way mutually compensating **choreographing** between θ(x,t), A(x,t), and ∂_μ together. We modify an A with $A' = A + \nabla\chi$ and $D = \nabla - ieA/\hbar$ with the result that this combined change effectively "prevents observability of un-measurables" (can't see absolute phase, can't see redundant changes to potentials, can't see symmetries – such as the SU(n) groups themselves). The quanta of interacting gauge fields are bosons such as the photon represented by the A field, gluons in QCD, and heavy bosons of electroweak theory.

In the case of Dirac QED, "the crucial point is that substituting $\partial_\mu \to D_\mu$ into the Dirac Lagrangian automatically gives the **interaction** $\mathbf{J^\mu A_\mu}$ term $\mathcal{L}_{int}$ which agrees with the "vertex" of quantum electrodynamics" Feynman diagrams. And $A' = A + \nabla\chi$ also leaves the important 4×4 electromagnetic $F^{\mu\nu}$ tensor unchanged in the electromagnetic or the Dirac Lagrangian. In addition, gauge symmetry plays a key role in making gauge theories "renormalizable" thus producing useful answers instead of infinity.

For those unfamiliar with saying that an interaction is "JA," we should refer to things you do know: Note that in simple vector form: $L_{classical} = mv^2/2 + \vec{A}\cdot q\vec{v} - e\phi$ has the "interaction" term $[\vec{A}\cdot\vec{J}_{single\ particle}]$ as a "velocity dependent potential." And plugging that into the usual "Lagrange-Euler equations" solution $\{\partial t(\partial L/\partial\dot{x}) = \partial L/\partial x\}$ automatically yields the all-important and more familiar "**Lorentz-Force Law," F = qv × B** + **qE**. {e.g., see https://en.wikipedia.org/wiki/Lorentz_force}. This is foundational for observing the effects of magnetic fields on moving charges. It should also seem initially strange in the sense of one velocity arrow crossed into another magnetic arrow yields a force perpendicular to both. {What other force does this? The Coriolis "fictitious" force due to being in the "wrong" frame of reference – that's a clue. It also occurs in general relativity with the 1918 Lense-Thirring dragging of inertial frames. And that in turn produces a "gravito-magnetic" effect like the Lorentz force (e.g., important for the case of rotating Kerr black holes).

These examples refer to large scale fields. But, also note that if two individual charged particles scatter off each other, they each contribute a "current" $e\vec{v} \simeq J$ that causes a mutual interaction A_μ that Zee calls $Z(J_1J_2)$ with "internal line" exchange in a Feynman diagram – "the photon." We could say that the charges radiate the gauge field, A.

Advancing to Yang-Mills theory is similar but much more complex because it moves into higher continuous gauge symmetry groups such as SU(2) and SU(3) which are "non-abelian" {elements uv ≠ vu}. These Lie groups represent the symmetry of the Lagrangian. We might picture angles as representing real rotations, but SU(2) is the group of rotations in complex two-dimensional space , $\mathbb{C}^2$. Its local internal symmetry might be called fictitious because unlike a global gauge symmetry it does not correspond to a conserved physical quantity.

"The essential building blocks of gauge theory are the gauge symmetry group, the gauge potential field which defines the connection, and the physical particles which are the sources of the gauge field and which also interact with each other via the gauge potential" [Moriyasu, p. 34]. Potential, A_μ, "is both an external field and an internal space operator." For the non-Abelian gauge groups, the "potential fields can carry internal charges" as well. U(1) is Abelian {meaning that the product of elements $U(\theta_1)U(\theta_2) = U(\theta_2)U(\theta_1)$ }, and that means that the photon doesn't self-interact.

How about also considering Dirac's gamma matrix algebra (which is a Clifford algebra, Cℓ(1,3)), Gell-Mann matrix algebra, and spinors as internal spaces. Feynman "slash" notation uses gamma matrices and presents as a 4-vector. Strong and electro-weak interactions also have "gauge groups" with continuous transformations that are valued in Lie groups. QCD is based on unbroken $SU(3)_{color}$; and electro-weak theory uses the combination SU(2)×U(1) with spontaneous symmetry-breaking (the "Higgs" mechanism). "The color group SU(3) corresponds to the local symmetry whose gauging gives rise to QCD [wik].

Some Problems:

A local change of phase by $e^{i\theta} = e^{iq\chi(x)/\hbar}$ is said to be compensated by $A' = A + \nabla\chi$ where gradient of chi is <u>really</u> useless ($\nabla\times\nabla\chi \equiv 0$). The field A = 0 satisfies this. How to justify introducing a background A(x,t)≠ 0 field that is useful? Maybe it is already a given as deduced from Maxwell equations. So, the early issue was what did we also already know about strong and weak forces? Initially, not much.

Putting $\partial_\mu^2 \rightarrow D_\mu^2$ into the Schrodinger equation gives an $ieA\cdot\nabla\psi$ term needed for current-field interactions but also an e^2A^2 term which breaks gauge invariance – how is this discarded? Note that the Dirac part of its own Lagrangian only has only one derivative $\partial_\mu \rightarrow D_\mu$ term which automatically leads to the proper interaction without an e^2A^2 term: $\mathcal{L}_{int} = q\bar{\psi}\,\gamma^\mu A_\mu \psi = J^\mu A_\mu$. So proper QED resolves the non-relativistic Schrodinger problem.
Gauge theory itself doesn't much discuss charges and currents – they are added on.

Suppose we play around a bit trying to take Chi(x,t) seriously:

First, consider it as a local "**bump**" in space-time – a "nice shape" is Gaussian (Bell)-- perhaps like $\chi(x) = B\exp[-(br)^2]$. Its derivative, $\nabla\chi = \hat{r}\,\partial\chi/\partial r = -2\hat{r}\,b^2 rB\exp[-b^2r^2]$ has to resemble a vector potential. Instead of peaking at the center like Bell, this is maximum in a spherical shell around center.

UNITS: Let **[ ] stand for "units of":** find [χ]=[B]: [A] = joules/amp-meter = volt-s/m = [momentum]/charge, and we wish $q\nabla\chi$ to be like a momentum [p]. So, [q χ] = [p]· meters, and [χ] = [p]meters/unit charge

And [b] = [1/r] so $[b^2r] = [1/r]$ and [B] = [r ∇χ]=[r ∂χ/∂r] = [χ], OK.

How about the other potential ∂χ/∂t in S.I. volts, so [χ(x,t)] =[ϕ] is volts-seconds = [A]· meters – so, at least this is consistent. But, how about an interpretation for this sub-potential χ?

What is the meaning of [qχ] = momentum × meters in every point in the "bump"?

Consider that [Planck h] = joules/hz = joules·sec = kg m^2/s = momentum times distance, ….is that interesting?

Well, we already knew that angle **$\theta = q\chi(x)/\hbar$,** so of course [qχ] = [h]. For q = electric charge, qχ drives matter wave radian angle changes in units of $\hbar$.

So, one question now is, "How real are the 'bumps' ?"

If we take the time derivative of the angle equation, we get $\Delta E = \hbar\omega = \hbar\,\partial\theta/\partial t = = q\partial\chi/\partial t = q\Delta V$ – and we have already heard suggestions that ramping up potential voltage increases the energy of a system. The possible existence of a "chi" field would only apply to the quantum world.

There is an electrostatic Aharonov-Bohm effect using potentials only without electric fields. It was partly verified in 1998 in a form like that above: $\boldsymbol{\hbar\,\Delta\theta/\Delta t = q\Delta V}$ where Δθ is a resulting electron phase change, ΔV is an exposure to a higher voltage and Δt is time spent traversing a distance in an increased potential. A thorough test is a very difficult experiment and has yet to be done.

Conclusion:

Inner spaces are hidden from us, and we have been thinking of them as apparently useful within tiny regions of space-time. Intuitively, it is somewhat like having new dimensions that are only seen by special "Inward Bound" close-up observers (but different from the ultra-tiny Planck size dimensions of string theory). Now, concepts like entanglement and Bell's tests suggest new arenas over large regions. Space-time is not a totally comprehensive arena. The universal fields for each "elementary particle" throughout all space-time is also of course large scale. And the Higgs potential field is universal and imparts mass to some particles.

Many of us believe in a reality of quantum waves and that the vector potential $\vec{\mathbf{A}}$ possesses at least some aspects of reality (and "qA" affects $e^{i\phi(x,t)}$). The foundation of quantum mechanics begins with particle mass being an ultra-high frequency rest-vibration ($h\nu = E = mc^2$) whose Lorentz transformation to a moving frame {relative velocity $\vec{v}$ } yields Schrodinger wavelengths **λ = h/p** as a purely relativistic phenomenon (in "*non-relativistic* quantum mechanics"). Redundancy in the possible values of wave phase and choice of A leads to exploration of a "chi" field as a "non-physical transformation of momentum": $\theta \simeq q\nabla\chi\,(x,t)$ with $\nabla\chi \cdot d\ell = d\chi$ implies that $\int \nabla\chi \cdot d\ell = \Delta\chi$ between two points, and $\Delta p = q\Delta\chi$. "Chi" is allowed to exist locally, and charge couples with chi to give a shift in momentum. It could happen; but does it happen?

Despite gauge freedom of choices, we might consider a natural preferred choice and intuitively pleasant interpretation of A as due to the effective "dragging of an electromagnetic space by moving charge currents." This is seen in the "Lienard-Wiechert potential" $A = (\mu_o/4\pi)\int (\vec{J}(r)/r)d^3r$ *{but, it does use a particular "Lorenz gauge" $\partial_\mu A^\mu = 0$}.* However, it is also a consequence of a Lorentz transformation to a moving frame of a simple Coulomb field from charge --meaning that "preferred A" is interpreted as just the dragging of the electrostatic field to a velocity v. That might suggest the concept of gauge freedom of the field as more of a strangely useful but unreal "mathematical game."

Supplementing the concept of "interior reality," philosopher of physics Ruth Kastner adds the phrase, "outside of space-time." A particular note is that the quantum amplitude "waves exist as possibilities outside of physical spacetime, and therefore it is necessary to accept such possibilities as part of reality [wik]." Potentia can be real. Nicolas Gisin adds, "quantum correlations are coming from outside space-time."

The following quote is interesting: "There must be a new way of thinking about quantum field theories, in which space-time locality is not the star of the show. . . . by removing spacetime from its primary place in our description of standard physics, we may be in a better position to make the leap to the next theory, where space-time finally ceases to exist. {Arkani-Hamed (2012), [Peebles] }.

Bottom Line: Gauge theory is a basic, essential, powerful, and revealing tool. But, it is also somewhat "magical;" and its {sub-} physical reality is far from apparent.

References:

[Heisenberg] W. Heisenberg, "Über den Bau der Atomkerne." Zeitschrift für Physik, Volume 77, pages 1–11, (1932) – and two others in a set of articles (accessed by Mike Jones).

[Tong] https://www.damtp.cam.ac.uk/user/tong/gaugetheory/2ym.pdf 2. Yang-Mills Theory

[Ait] I.J.R. Aitchison & A.J.G. Hey, Gauge Theories in Particle Physics, 2nd Ed. Adam Hilger, 1989.

[**Renaud**] Pierre Renaud, "Clifford Algebras Lecture Notes on Applications in Physics," 253 pages 2020-2022. https://hal.science/hal-03015551v2/file/The_Clifford_algebra_book-24%20%281%29.pdf

[Moriyasu] K. Moriyasu, An Elementary Primer for Gauge Theory, World Scientific, 1983.

[ZeeNut] A. Zee, Quantum Field Theory in a Nutshell, Princeton, 2003,

[Zee-simply] A. Zee, Quantum Field Theory as Simply as Possible, Princeton, 2023.

[Tong] https://www.damtp.cam.ac.uk/user/tong/gaugetheory/2ym.pdf 2. Yang-Mills Theory

{Yang-Mills is built upon the mathematical structure of Lie groups and is a very non-classical strongly-coupled quantum field theory}.

[Shifman] Misha Shifman (Particle Physics high Energy Physics) QFT II 318 pages https://www.worldscientific.com/doi/pdf/10.1142/9789813234192_0001

[Peebles] P. J. E. Peebles The physicists philosophy of physics https://arxiv.org/pdf/2401.16506.pdf 46 pages .

[Auyang] Sunny Y. Auyang, How is Quantum Field Theory Possible? Oxford, 1995 [physics406] https://www.pas.rochester.edu/~rajeev/phy406/Symmetries13.pdf Yang-Mills Theory is the foundation of the theory of elementary particles.

[Georgi] Howard Georgi, Lie Algebras in Particle Physics; From Isospin to Unified Theories; Second Edition 339 pages, 1999, 2019
https://library.oapen.org/bitstream/handle/20.500.12657/50876/9780429967764.pdf?sequence=11&isAllowed=y
[Stoica] https://iopscience.iop.org/article/10.1088/1742-6596/880/1/012053/pdf "The Standard Model Algebra - a summary," Ovidiu Cristinel Stoica, 2017,.
[dp2013] Dave Peterson, "Gauge Theory for Electromagnetism," in A Stroll Through Physics, 2014, book or web www.sackett.net/DP_Stroll.pdf .
[Kunasz] Lie Groups, http://www.sackett.net/LieGroupsInPhysics.pdf
[J.D.] https://www.fuw.edu.pl/~derezins/clifford.pdf Clifford algebras and fermions Jan Dereziński 86 pages As usual, instead of representations of Lie groups, we will usually speak about representations of the corresponding Lie algebras
[Healey] Richard Healey, "On the Reality of Gauge Potentials," {44 pgs} https://philsci-archive.pitt.edu/328/1/RLGAUG%2Bfiguresfinal.pdf Philosophy Department, University of Arizona

Notes: On the reference [Renaud] Pierre Renaud, "Clifford Algebras Lecture Notes on Applications in Physics," 253 pages 2020- 2022. https://hal.science/hal-03015551v2/file/The_Clifford_algebra_book-24%20%281%29.pdf It looks like a great book! Discussions ≥ pg 82 are important, study!

[Arxiv]: https://arxiv.org/pdf/2401.05678.pdf enlisting the aid of the Clifford algebra approach, whereby both the algebraic spinor states and Dirac's gamma operators can be expressed in the same algebraic space.
Note The $U(1) = U(1)_Y$ occurring in the standard model is actually the "weak charge," a combination of electric charge and isospin