Chaos Questions/Answers from Cosmo Zoom Meeting of 7/17/22: dave peterson

Thus speaketh the Google.

Is Chaos a Science?

"Science is a systematic enterprise that builds and organizes knowledge in the form of testable explanations and predictions." But, "Chaos Theory deals with nonlinear things that are effectively impossible to predict or control, like turbulence" [https://plato.stanford.edu/entries/chaos/]. It is then "less of a new science than a progression in thinking, a shift in world views, from Newtonian determinism to nonlinear unpredictability." "The kind of understanding provided by chaos models is challenging to clarify," and "dynamic understanding" appears to be descriptive only" – it explains patterns. There is no consensus among mathematicians and physicists for any precise definition of chaotic behavior, As mentioned by Chela, the name "chaos" is now a topic mainly under the umbrella of "nonlinear systems analysis." For example, the daily articles on-line archive for Physics ArXiv.org has a **Nonlinear Sciences** heading that includes: Adaptation and **Self-Organizing Systems**; Cellular Automata and Lattice Gases; **Chaotic Dynamics;** Exactly Solvable and Integrable Systems; Pattern Formation and Solitons. And "Math ArXiv" includes Dynamical Systems.

Heavy Tails and Power Laws:

Gleick (pg. 84) mentioned that jumps in Cotton prices had a "long tail" distribution (they fall off much more slowly than the tail of a Bell curve). Mandelbrot (1963) considered this as a "**Pareto**" or power-law heavy tail (probability density $dP/dx = A/x^p$ or "log-log" straight line) which is relatively "scale-free" (large and small scales look about the same). That is a Fractal property (although fractals can also be a separate topic). "Zipf's Law" is another frequently encountered power law example (https://en.wikipedia.org/wiki/Zipf%27s_law). Gleick also referred to electrical noise and Nile elevation examples (pg 92). Commonly encountered Fractals and power-law statistics in the real world have limited ranges that may only work over two to five orders of magnitude, and they are "random" rather than regular fractals. They truncate at some big and some small size limit. An ideal fractal goes on forever, and the term prefractal can be used when self-similar structure exists on only a finite number of length scales. For certain values of the power slope, standard-deviations or even means may not exist! (meaning that these calculated measures aren't stable and will increase without bound as more and more data is included). An outcome of "power law is one of the common signatures of a nonlinear dynamical chaotic process."

Dimension: *Can attractors also have a dimension? Yes.*

For Fractals: *Gleick p. 100*, every iteration of the Koch star multiplies the total length by 4/3rds. The dimension is then $\log_3 4 = \log 4/\log 3 \simeq 1.26 > 1$ *(whether base 10, 2, or e doesn't matter—it cancels out).* The length is $L_{n\ (\text{from 0 to n})} = 3\cdot(4/3)^n$ which will approach ∞. For the Cantor dust (p 93), d is $\log 2/\log 3 \simeq 0.63 < 1$. The dimension of many other fractals is given in https://en.wikipedia.org/wiki/List_of_fractals_by_Hausdorff_dimension . Notice that dimensions are also given for some attractors such as the Feigenbaum attractor (d = 0.538) and the Lorentz attractor (d = 2.06 –"measured"—or "box counting" calculation – having a non-integer dimension makes this attractor "strange"). The definition of the dimension of an attractor is the same as for a fractal but computation is more difficult (in a surrounding lattice, is a point of an attractor in a box or not? -- and the box can be a cube for 3d).

RUNAWAY STARS and Chaos: *Bill Daniel comment.*

It is known that star clusters can eject stars and black holes with speeds above 30 km/sec; and backtracking known trajectories often does implies a globular cluster source (GCs). Some of the "evaporated" ejected stars are "due to unstable orbital configurations of stellar groups" (and here we ignore effects from supernova explosions, winds and mega-black holes). It is not clear that Chaos applies broadly to GCs, but three-body simulations do often result in an ultimately ejected body. Lighter stars are easier to eject." When the three bodies have 10-sun *(10 times the mass of the sun),* 15-sun and 20-sun masses, for instance, the 10-sun star gets kicked out in 78% of the simulations." "The velocity acquired by the ejected star easily exceeds the escape speed of a star cluster."

Quantum Chaos ? :

Chaos only exists in nonlinear systems, but the Schrodinger equation is **linear**. Adding interactions can make quantum mechanics nonlinear. We've studied deterministic chaos, but quantum mechanics is indeterministic *{ however "there is a serious open question as to whether the indeterminism in quantum mechanics is simply the result of ignorance due to epistemic limitations or if it is an ontological feature of the quantum world."}* "There is the thorny problem of defining quantum chaos. The difficulties in establishing an agreed definition of quantum chaos are actually more challenging than for classical chaos."

This is an active field and books have been written on the topic. "Quantum chaos is the study of the quantum dynamics of systems that are classically chaotic. Contrary to the impression left by most textbooks, almost all conservative dynamical systems are at least partly chaotic in the range of their behavior."

There are some new restrictions for quantum chaos. e.g., "the notion of Lyapunov exponents does not directly carry over to the quantum regime," and there are no traditional trajectories. "Bounded, fully quantized systems that exhibit exponential sensitivity and infinite recurrence are genuinely quantum chaotic" -- "the existence of systems that are quantum chaotic …remains an open question."

This sounds like a preliminary work in progress that doesn't seem very inspiring so far (to me).

COMPLEXITY

We would like to think that Self Organization, Emergence and "More is Different" are inter-related. There is a nice book on this: **MORE AND DIFFERENT, Philip Anderson,** World Scientific, 2011, 412 pages, paperback, *could skip p 263-350.* (beyond reductionism, wholes that are more than the sum of their parts). $38.

In addition, there is now a book **Why More is Different**
[Look Ahead here on one of Rick's suggested books] Why More Is Different, 2015, $70-90 [Unfortunate Price. Note Amazon does not provide these details or a Look Ahead].
https://www.google.com/books/edition/Why_More_Is_Different/fXjdBgAAQBAJ?hl=en&gbpv=1&printsec=frontcover

Philosophical Issues in Condensed Matter Physics and Complex Systems
Editors: Brigitte Falkenburg, Margaret Morrison

A unique look at the surprisingly many philosophical questions pertinent to condensed matter physics -- Tackles the key issues of reduction, emergence and reversibility as they relate to various condensed matter phenomena

[Wikipedia] "**Complexity** characterises the behaviour of a system or model whose components interact in multiple ways and follow local rules, leading to nonlinearity, randomness, collective dynamics, hierarchy, and emergence. The term is generally used to characterize something with many parts where those parts interact with each other in multiple ways, culminating in a higher order of emergence greater than the sum of its parts.

https://en.wikipedia.org/wiki/Complexity {also includes self organization, nonlinear dynamics and pattern formation} https://en.wikipedia.org/wiki/Complex_system

The famous MORE IS DIFFERENT original SCIENCE essay article 1972 by P.W. Anderson at https://cse-robotics.engr.tamu.edu/dshell/cs689/papers/anderson72more_is_different.pdf **or**

https://www.tkm.kit.edu/downloads/TKM1_2011_more_is_different_PWA.pdf