Quantum electrodynamics based on self-fields: On the origin of thermal radiation detected by an accelerating observer

A. O. Barut and Jonathan P. Dowling*
Department of Physics, Campus Box 390, University of Colorado, Boulder, Colorado 80309
(Received 7 April 1989; revised manuscript received 2 October 1989)

We continue with our series of papers concerning a self-field approach to quantum electrodynamics that is not second quantized. We use the theory here to show that a detector with a uniform acceleration a will respond to its own self-field as if immersed in a thermal photon bath at temperature $T_a=\hbar a/2\pi kc$. This is the celebrated Unruh effect, and it is closely related to the emission of Hawking radiation from the event horizon of a black hole. Our approach is novel in that the radiation field is classical and not quantized; the vacuum field being identically zero with no zero-point energy. From our point of view, all radiative effects are accounted for when the self-field of the detector, and not the hypothetical zero-point field of the vacuum, acts back on the detector in a quantum-electrodynamic analog of the classical phenomenon of radiation reaction. When the detector is accelerating, its transformed self-field induces a different back reaction than when it is moving inertially. This process gives rise to the appearance of a photon bath, but the photons are not real in the sense that the space surrounding the accelerating detector is truly empty of radiation, a fact that is verified by the null response of an inertially moving detector in the same vicinity. The thermal photons are in this sense fictitious, and they have no independent existence outside the detector.

I. INTRODUCTION

In the wake of the discovery by Hawking of the apparent thermal emission from the event horizon of a black hole,[1] there came a related calculation by Unruh[2] that indicated that a uniformly accelerating particle detector would perceive a thermal bath of photons. If an idealized point detector is accelerating at a rate a, then the photon spectral distribution is Planckian at a temperature $T_a=\hbar a/2\pi ck$, where k is the Boltzmann constant. This thermal radiation is not picked up by an inertially moving detector, and the vacuum expectation of the normal ordered stress energy tensor $T_{\mu\nu}$ is identically zero in both the inertial and accelerated or unprimed and primed frames, respectively;[3] $\langle 0|{:}T_{\mu\nu}{:}|0\rangle=\langle 0|{:}T'_{\mu\nu}{:}|0\rangle=0$. In what sense then can one say that these thermal photons are physically real if they do not alter the above expectation values? Davies argues that these results are indicative of a breakdown of the traditional quantum field theoretical notion of a particle when space-time is curved.[3] The present authors contend that the problem is not with the concept of particle but rather with the quantum field treatment of the vacuum field. Boyer has given an account of the Unruh effect in the framework of stochastic electrodynamics, which lends credence to the viewpoint that the acceleration somehow turns the virtual quanta of the Minkowski vacuum into real quanta.[4] In stochastic electrodynamics the zero-point field is taken to be a very real thing, responsible for many quantum-electrodynamical phenomena. The idea is that a classical vacuum with a spectrum proportional to $\frac{1}{2}\hbar\omega$ per normal mode is permissible on the grounds of Lorentz invariance. If one chooses the proportionality constant appropriately, one recovers a classical vacuum field that is nearly identical to that predicted by the second quantization procedure in field theory. Boyer then shows that under acceleration, the zero-point term is deformed into a zero-point plus Planckian spectrum at the Unruh temperature $T_a=\hbar a/2\pi kc$. The transformation is

$$\hbar\omega\rightarrow\hbar\omega\left[\frac{1}{2}+\frac{1}{e^{\hbar\omega/kT_a}-1}\right], \tag{1}$$

where we will from now on set $\hbar=c=a=1$.

But are these thermal photons really real? Indeed, one may ask if even the virtual Minkowski photons with the spectrum of $\frac{1}{2}\hbar\omega$ have any real existence apart from the detector that appears to register them, say, as the apparent "trigger" for spontaneous emission. In stochastic electrodynamics the choice of a nonzero proportionality constant for the spectrum proportional to $\frac{1}{2}\hbar\omega$ is permissible, but not required. The other obvious choice is to set the spectrum of the vacuum identically equal to zero as is done in classical electrodynamics. Where then would radiative effects such as spontaneous emission and the Lamb shift originate if not driven by the vacuum fluctuations, as is usually assumed in quantum electrodynamics (QED)? In classical electrodynamics there are two perfectly respectable phenomena which should correspond to the classical limit of spontaneous emission and the Lamb shift in atoms; they are line breadth and level shift in the energy, for instance, of a harmonically bound charge.[5] These radiative corrections to the otherwise unperturbed motion of the charge arise not from any interaction with a zero-point field—the classical vacuum field is identically zero—but rather from the radiation reaction on the charge from its own self-field. The scale of the electromagnetic field fluctuations is set by the constant $\hbar c$, so in the classical limit of $\hbar\rightarrow 0$, one would expect spontaneous emission and the Lamb shift to vanish and to have no classical analog since the causative agent, the zero-point field, has vanished. This is clearly not the case in that we are actually left with the classical line breadth and level shift of an oscillating charge. Barut and his co-workers have shown that it is possible to formulate QED in terms of self-fields, so that such phenomena as spontaneous emission and the Lamb shift are viewed as natural generalizations of their classical counterparts in radiation reaction theory.[6] This is the approach that we shall use in the present work.

In QED it is usual to renormalize the free electromagentic field through the normal ordering of the operators in order that the zero-point energy of $\frac{1}{2}\hbar\omega$ per normal mode vanishes. This is done primarily because the keeping of the $\frac{1}{2}\hbar\omega$ in the Hamiltonian would lead to an infinite energy density of empty space since $\langle 0|T_{\mu\nu}|0\rangle$ would diverge. The rationalization usually given for this procedure is that only energy differences have physical meaning, and hence a transfinite translation of the energy scale cannot have physical consequences. But the energy density $T_{\mu\nu}$ does indeed have an absolute meaning when coupled to the gravitational field, in the sense that it determines the curvature of the metric via the Einstein field equations. It is not possible to change the curvature or to flatten out space-time simply by adjusting the energy scale. If we accept the electromagnetic zero-point energy as real, then by implication we must accept the infinite energy density of empty space. This implies an infinite curvature for the universe and infinite value for the cosmological constant—unless we are saved in some unforeseen fashion by a fortuitous cancellation of all the vacuum fields in some unified field theory. The cosmological constant Λ is the most accurately determined physical quantity in all of physics; the observations by Sandage[7] of distant galaxies puts $|\Lambda|=0$ with an upper limit of $|\Lambda|<10^{-56}\ \mathrm{cm}^{-1}$. It certainly is not infinite.

It is common to say that the vacuum fluctuations are the physical cause of spontaneous emission, the Lamb shift, the nonzero value of $g-2$, the Casimir effect, long-range Casimir-Polder van der Waals forces, apparatus dependent contributions to these radiative effects, and now the thermal response of an accelerating detector. This view is perhaps that of the majority. It is not as well appreciated that all of these effects may be equally well explained at least to order α, in terms of the fields which originate in the charged particles themselves.[6,8]

Jaynes[9] has given a nice example of why the zero-point fluctuation interpretation of radiative effects in QED makes many of us uneasy. Suppose we believe that the electromagnetic zero-point energy is physically real, right up to the Compton cutoff frequency $\omega=m$, which is used in nonrelativistic calculations of the Lamb shift to get the correct Bethe logarithm. If one computes the turbulent energy flow associated with the zero-point field at this cutoff, one gets a Poynting vector of about 6×10^{20} $\mathrm{MW\,cm}^{-2}$. (The total luminosity of the sun is about 2×10^{20} MW.) One feels that physically real radiation of this intensity would have slightly more of an effect than to shift the 2s level of hydrogen by 4 μV.

Much work has been done in the past few years to show that there is a deep and fundamental connection between the vacuum fluctuation and the self-field approaches to QED.[10] The duality between these two methods of doing QED does not necessarily prove, however, that the zero-point field is real. It is possible that self-field effects are the same *as if* vacuum fluctuations were the causative agent. Jaynes has shown that the energy density of the radiation reaction field over the spectral interval of the natural linewidth is exactly the same as that of the vacuum field.[9] In the present paper we will support this idea that the vacuum field approach is a mathematical subterfuge which gives the correct answer some of the time.

Davies has emphasized that the meaning of the concept of a particle and the codependent concept of the vacuum depends crucially on the state of motion and history of the particle detector.[3] This is a fact which is often overlooked in Minkowski space, but which cannot be ignored in curved space-time where the decomposition of the field into positive and negative frequency normal modes is not unique for all observers. In general, different detectors will disagree on what constitutes the vacuum. If one detector registers no particles, a different detector on a different world line, in general, *will* register particles. This is because a Bogoliubov transformation between the two Fock spaces used to define the vacuum for each detector will not give identical vacuums for the two spaces. Davies concludes that because of this the concept of a particle, say a photon in the electromagnetic case, is not well defined. The present authors would like to use the same evidence to support a different conclusion: It is the standard notion of the vacuum in quantum field theory that is not well defined, a fact which seems obvious when one begins to consider quantum fields in curved space.

The stochastic electrodynamics theory of Boyer also develops pathological problems in curved space-time. Boyer chooses a classical zero-point spectrum proportional to $\frac{1}{2}\hbar\omega$ per normal mode because this is the only nonzero spectrum permitted on the grounds of Lorentz invariance. This means that in Minkowski space the stochastic electrodynamic vacuum is permitted since it is an invariant of the Poincaré group. The Poincaré group is not a symmetry group of a general curved space-time, however, and apparently the most compelling reason for choosing a stochastic zero-point spectrum proportional to $\frac{1}{2}\hbar\omega$, rather than to zero, completely disappears. The only choice of such a proportionality constant consistent with the demands of a space-time of arbitrary curvature is one that is zero. By Boyer's own reasoning we must conclude the only allowable classical vacuum field in curve space-time is the same as that used in classical electrodynamics—namely, the vacuum field must be chosen to be identically equal to zero in all its moments of the Wightman correlation function.

In discussing quantum fields in curved space-time the concept of a detector plays a central role: It is impossible to discuss properties of the quantum vacuum field in the absence of a detector to observe those properties. One cannot speak of the absence or presence of a vacuum without a detector to register deviations or nondeviations from that vacuum state. The concept of a vacuum in the absence of a detector is meaningless in both a philosophical and operational sense. But by the definition of a detector, it must couple to the vacuum field whose presence or absence we are trying to measure, and hence introduce its own self-field into the measurement process.

In the present paper we shall show that the self-field of a uniformly accelerating point detector responds to the acceleration in such a way as to drive the detector, via a quantum generalization of radiation reaction, into a superposition of states which when thermodynamically analyzed yields the Planck spectrum given in Eq. (1). But now the interpretation is different. The $\frac{1}{2}\hbar\omega$ corresponds to the spectral distribution of the detector's own field over the natural linewidth. For an inertially moving detector this is the only term which occurs, and it is responsible for the usual free-space atomic spontaneous emission as well as for the Lamb shift. Transforming to the Rindler coordinates of a uniformly accelerating detector we obtain the full result of Eq. (1). The interpretation now is *not* that the detector is immersed in a bath of thermal photons, but rather that the self-field of the detector is responding to the work being done on it by the accelerating agent in such a fashion so as to make it appear *as if* the detector were immersed in such a bath. There are no physical photons present, a fact which would be confirmed by a neighboring inertially moving detector. The particle concept is hence rescued, but only with the sacrifice of the notion of a dynamic vacuum state in the absence of a detector. Since one cannot discuss the vacuum without the detector, it would seem compelling to want to set the vacuum field identically equal to zero for all observers and then to attribute differences between detectors totally to the response of the self-field of the detector to its own worldline.

Notice that by the Einstein equivalence principle, a uniformly accelerating detector is equivalent to a detector at rest in a gravitational field. From the point of view of general relativity the thermal radiation seen by the detector seems to originate from a neighboring event horizon; in our calculation it would be the event horizon of Rindler coordinates. But this event horizon is related to that of a black hole by a conformal transformation. Hence Unruh and Hawking radiations are similar, and from the self-field point of view they are both equally interpreted in the sense that the thermal radiation effects are confined to within the detector, which is responding to the gravitational field directly.

II. SELF-FIELD APPROACH TO QED

In classical electrodynamics one usually computes the zeroth-order motion of the charges first and then adds on the self-field or radiation reaction effects as a perturbation to the original motion. (Although, in special circumstances, one may find exact solutions to the nonlinear Lorentz-Dirac equation of motion.) The philosophy in the self-field approach to QED is precisely the same. Conceptually we may separate the electromagnetic four-potential A_μ surrounding a point charge into and external field A^e_μ, which is prescribed as part of the initial conditions, and a self-field term A^s_μ, which originates from the charge. The total field is then $A_\mu := A^e_\mu + A^s_\mu$. (The notation $a := b$ indicates that a is being defined as being equal to b with the colon on the same side as the quantity to be defined.) With this separation, we shall assume that the coupling of the Dirac spinor field Ψ to A^e_μ alone determines the bulk, zeroth-order motion of the electron, while the coupling to A^s_μ is responsible for all radiative corrections. As in previous work we find it convenient to proceed from the standpoint of an action formalism,[6,8] with the action given by

$$W = \int dx\, w[x;\Psi;A] \;, \tag{2}$$

where W is the total action and $w(x)$ is the corresponding action density. Variation of Eq. (2) with respect to Ψ yields the Dirac equation of motion, while variation with respect to A_μ gives Maxwell's equations. The electromagnetic field tensor $F_{\mu\nu}$ is defined as usual as

$$\begin{aligned} F_{\mu\nu} &:= A_{[\nu,\mu]} \\ &= A^e_{[\nu,\mu]} + A^s_{[\nu,\mu]} \\ &=: F^e_{\mu\nu} + F^s_{\mu\nu} \;, \end{aligned} \tag{3}$$

where $[\,,]$ indicates commutation with respect to the indices. Far from the source of the external field we have Maxwell's equations as

$$(F^{\mu\nu})_{,\mu} = (F^{\mu\nu}_s)_{,\mu} = ej^\nu \;, \tag{4}$$

where j_μ is the usual Dirac current $\Psi\gamma_\mu\Psi$. The action density $w(x)$ can be written now as

$$\begin{aligned} w &= \overline{\Psi}(i\gamma^\mu\partial_\mu - m)\Psi + eA_\mu j^\mu + \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &=: w_0 + w_1 + w_2 \;, \end{aligned} \tag{5}$$

where w_0 is the free particle density, w_1 the particle-field coupling, and w_2 the free electromagnetic (EM) field action density. It is evident that w_0 and w_1 taken together are equivalent to the canonical coupling $i\partial_\mu \rightarrow i\partial_\mu - eA_\mu$. At this point the external and self-electromagnetic field have not yet been separated. We proceed now with an analysis of w_2,

$$w_2 = \tfrac{1}{4}(F^e_{\mu\nu}F^{\mu\nu}_e + F^e_{\mu\nu}F^{\mu\nu}_s + F^s_{\mu\nu}F^{\mu\nu}_e + F^s_{\mu\nu}F^{\mu\nu}_s) \;. \tag{6}$$

The two middle terms can be converted into surface integrals under $\int dx$, which vanish if A^s_μ is sufficiently localized. The first term of this expression is the invariant

$$\tfrac{1}{4}F^e_{\mu\nu}F^{\mu\nu}_e = -\tfrac{1}{2}(E_e^2 - B_e^2) \;, \tag{7}$$

which is an additive constant that does not effect the equations of motion, and so we may drop it from the action. We are left with the last term, which can be transformed as

$$\begin{aligned} \tfrac{1}{4}F^s_{\mu\nu}F^{\mu\nu}_s &= \tfrac{1}{4}A^s_{[\nu,\mu]}F^{\mu\nu}_s \\ &= \tfrac{1}{4}(A^s_{[\nu}F^{\mu\nu}_s)_{,\mu]} - \tfrac{1}{4}A^s_{[\nu}(F^{\mu\nu}_s)_{,\mu]} \\ &= -\frac{e}{2}A^s_\nu j^\nu \;, \end{aligned} \tag{8}$$

where we have used the inhomogeneous Maxwell equations (2). Equality here is with respect to integration by parts and the vanishing of possible surface integrals under the application of $\int dx$. (Surface terms are, however, needed in the discussion of processes in which radiation goes to infinity such as in bremsstrahlung, the Compton effect, etc.) With these results the total density of expression (5) becomes

$$w=\bar{\Psi}(i\gamma^\mu\partial_\mu-m)\Psi+eA_\mu^e j^\mu+\frac{e}{2}A_\mu^s j^\mu$$
$$=:w_0+w_i+w_s \ . \tag{9}$$

Together w_0+w_i are responsible for the zeroth-order motion of the electron in the external field, while w_s induces radiative corrections to that motion.

One may formally solve Eq. (4) for A_μ^s in terms of the current j_μ through the use of an electromagentic Green's function $D_{\mu\nu}$ via

$$A_\mu^s(x)=e\int dy\, D_{\mu\nu}(x-y)j^\nu(y) \ . \tag{10}$$

If we define $W_s:=\int dx w_s(x)$ as the contribution to the total action W from the self-field correction, then inspection of expressions (9) and (10) yields immediately that

$$W_s=\frac{e^2}{2}\int\int dx\, dy\, j^\mu(x)D_{\mu\nu}(x-y)j^\nu(y) \ . \tag{11}$$

This single nonlinear addition to the usual action contains information about all radiative effects, e.g., spontaneous emission, the Lamb shift, and the electron $g-2$ value.[6,8] The interpretation here is, once again, that these radiative corrections arise as an effect of the back reaction of the self-field on the motion of the electron in a manner analogous to the classical phenomenon of radiation reaction.

To insure the boundary conditions that provide the correct combination of retarded and advanced potentials, we choose for the Green's function $D_{\mu\nu}(x-y)$ the causal Feynmann propagator $D_{\mu\nu}(x-y):=-\eta_{\mu\nu}D_{\mu\nu}(x-y)$, where $\eta_{\mu\nu}$ is the Minkowski metric tensor with signature $(+---)$ and $D_{\mu\nu}(x-y)$ has the equivalent forms

$$D(x-y)=\frac{i}{4\pi^2}\left[\frac{1}{(x-y)^2}-i\pi\delta((x-y)^2)\right] \tag{12a}$$

$$=\frac{i}{4\pi^2}\left[\frac{1}{(x-y)^2+i\epsilon}\right] \tag{12b}$$

$$=\frac{1}{(2\pi)^4}\int dk\frac{e^{-ik(x-y)}}{k^2+i\epsilon} \tag{12c}$$

in the Feynman gauge.

Now in order to further analyze W_s in Eq. (11) we perform a Fourier expansion of the Ψ in terms of quasi-bound-state energies E_n, which are to be determined, since we anticipate using a bound electron as our Unruh detector. The expansion is

$$\Psi(x)=\sum\!\!\!\!\!\!\int_n \Psi_n(\mathbf{x})e^{-iE_nt} \ , \tag{13}$$

where the sum runs over positive and possibly negative energy levels. To a first iteration we assume that the Ψ_n with associated eigenvalues E_n exactly minimize the action W_0+W_i. We are then using these zeroth-order wave functions to evaluate the W_s radiative correction term in an iterative fashion. Inserting expression (12c) for the Green's function into the expression (11) for W_s; expanding each of the Ψ as per Eq. (13); and carrying out the dx_0, dy_0, and dk_0 integrations yields

$$W_s=-\frac{e^2}{2}(2\pi)^4\sum_{n,m,p,q}\int\int\int dx^3dy^3dk^3[\bar{\Psi}_n(\mathbf{x})\gamma_\mu\Psi_m(\mathbf{x})]$$
$$\times[\bar{\Psi}_p(\mathbf{y})\gamma^\mu\Psi_q(\mathbf{y})]$$
$$\times\frac{e^{ik\cdot(x-y)}}{\omega_{nm}^2-|\mathbf{k}|^2+i\epsilon}$$
$$\times\delta(\omega_{nm}+\omega_{pq}) \ , \tag{14}$$

where $\omega_{nm}:=E_n-E_m$. The δ function can be satisfied by either of the two choices

$$n=m, \quad p=q \tag{15a}$$

$$n=q, \quad m=p \ . \tag{15b}$$

The condition (15a) leads to a vacuum polarization term,[6] and we shall not consider it here. The condition (15b) leads to spontaneous emission and the Lamb shift, here interpreted as quantum analogs of the classical radiation reaction effects of line broadening and level shift.[6] We will consider in the present work how these phenomena are effected by boosting the detector into an accelerating frame.

III. RESPONSE OF AN INERTIALLY MOVING DETECTOR TO ITS SELF-FIELD

To illustrate the self-field method of approach we will now confirm that a pointlike detector on an inertial worldline experiences at zero temperature only the effects of the usual spontaneous emission and Lamb shift in free space, which occur via the interaction of the detector with its own field. (This is, of course, clear form the Lorentz invariance of the theory, but it is instructive as an illustrative example of how to apply the self-field approach to this kind of problem.)

The trajectory for an inertial detector can be written as

$$\mathbf{x}=\mathbf{x}_0+\mathbf{v}t=\mathbf{x}_0+\mathbf{v}\gamma\tau \ ,$$
$$\mathbf{y}=\mathbf{y}_0+\mathbf{v}u=\mathbf{y}_0+\mathbf{v}\gamma\nu \ , \tag{16}$$

where τ and ν are the proper times which correspond to the x_μ and the y_μ time components $t=\gamma\tau$ and $u=\gamma\nu$, respectively. The velocity $\mathbf{v}$ is constant, with $v:=\beta<1$ and $\gamma:=(1-\beta^2)^{-1/2}$ as usual. The coordinates $\mathbf{x}_0$ and $\mathbf{y}_0$ are those of the electron in a detector based system. For a pointlike detector we may take $\mathbf{x}_0-\mathbf{y}_0\approx\mathbf{0}$ or $\exp[\mathbf{k}\cdot(\mathbf{x}_0-\mathbf{y}_0)]\approx 1$ in the dipole approximation.

Inserting the expressions (16) into the Green's function of (12b), we first notice that

$$(x-y)^2+i\epsilon=(\xi+i\epsilon)^2 \ , \tag{17}$$

where we have defined $\xi:=\tau-\nu$, and we have absorbed a positive function of ξ into the ϵ. The self-field action of Eq. (11) can now be written as

$$W_s^{\text{inertial}} = -\frac{e^2}{8\pi^2}\sum_{n,m}\int\int d\tau\, d\nu \frac{\langle n|\gamma_\mu|m\rangle\langle m|\gamma^\mu|n\rangle}{(\xi+i\epsilon)^2} \times e^{i\omega_{nm}\xi} , \quad (18)$$

where we have adopted the Dirac bra ket notation, and used the relation (15b) after expanding the Ψ as per the prescription of Eq. (13). The action W is formally infinite, but it can be related to the transition probability G_n for the nth energy level via[6,8]

$$W = 2\pi\delta(\omega_{nm})G_n ,$$

and the identification $\int\int d\tau\, d\nu \to \int d\xi$ gives us the finite transition probability per unit time for this nth state as

$$G_n = \frac{\alpha}{2\pi}\sum_m \langle n|\gamma_\mu|m\rangle\langle m|\gamma^\mu|n\rangle(\tfrac{1}{2}\omega_{nm}) . \quad (19)$$

In our units $\alpha = \theta^2/4\pi$, and the contour integral was carried out on an infinite semicircle in the lower ξ plane. A single pole of order 2 located at $\xi = -i\epsilon$ contributes

$$\tfrac{1}{2}2\pi i \text{ res}\left[\frac{e^{i\omega_{nm}\xi}}{(\xi+i\epsilon)^2}\right] = -2\pi(\tfrac{1}{2}\omega_{nm}) ,$$

where we note that if n is the ground state then $\omega_{nm} < 0$. A detailed analysis of (19) shows that this corresponds to the usual spontaneous-emission transition rate and Lamb shift in free space.[6] Notice how the factor of $\frac{1}{2}\hbar\omega_{nm}$ enters here, not as a consequence of any electromagnetic zero-point energy, but rather through the Fourier spectrum of the detector's self-field. Once again it looks *as if* there is a vacuum field which is stimulating the spontaneous emission or Lamb shift; in reality it is the detector's own field that is responsible.

IV. RESPONSE OF A UNIFORMLY ACCELERATING DETECTOR TO ITS SELF-FIELD

In the self-field approach to QED spontaneous emission occurs as a back reaction of the field on the detector. In curved space-time, such as in the Rindler coordinates of a uniformly accelerating observer, one would expect the self-field to become modified by the curvature and by a non-Minkowskian event horizon. Any change in the configuration of the self-field would be transmitted through the radiation reaction effects and would surface as a modification of the spontaneous emission rate, as well as of other radiative effects. We now compute this, the Unruh effect, from the self-field point of view.

Let us suppose that our detector is accelerating uniformly with acceleration $a := 1$. The worldline is hyperbolic and can be written in Rindler coordinates as

$$x_0 =: t = \sinh(\tau),\quad y_0 =: u = \sinh(\nu) ,$$
$$x_3 = \cosh(\tau),\quad y_3 = \cosh(\nu) , \quad (20)$$
$$x_1 = x_2 = 0,\quad y_1 = y_2 = 0 ,$$

with τ and ν the proper times as before. Hence we have, in the dipole approximation,

$$(x-y)^2 + i\epsilon = 4\sinh^2\left[\frac{\xi}{2} + i\epsilon\right] = -4\sin^2\left[\frac{i\xi}{2} - \epsilon\right] . \quad (21)$$

Once again we have absorbed a positive valued function of ξ into the ϵ. If we now insert expression (21) into Eq. (12b) for the Green's function we get

$$D(x-y) = -\frac{1}{16\pi^2}\csc^2\left[\frac{i\xi}{2} - \epsilon\right] . \quad (22)$$

Making use of the Laurent expansion for cosecant, namely,

$$\csc^2(z) = \sum_{p=-\infty}^{\infty} \frac{1}{(z-\pi p)^2} , \quad (23)$$

we can expand the Green's function of (22) as

$$D(x-y) = \frac{1}{4\pi^2}\sum_{p=-\infty}^{\infty}\frac{1}{(\xi + 2\pi i p + i\epsilon)^2} . \quad (24)$$

If we use this expression for the Green's function in the self-field action of (11) we eventually arrive at

$$W_s^{\text{accelerating}} = -\frac{\alpha}{2\pi}\sum_{n,m}\langle n|\gamma_\mu|m\rangle\langle m|\gamma^\mu|n\rangle \times \sum_{p=-\infty}^{\infty}\int\int d\tau\, d\nu \frac{e^{i\omega_{nm}\xi}}{(\xi+2\pi i p + i\epsilon)^2} , \quad (25)$$

with $\xi := \tau - \nu$ as before. Converting, as in the inertially moving case, to the transition probability per unit time per energy level n, and carrying out the integral on the same contour as before, we get

$$G_n = \frac{\alpha}{2\pi}\sum_m \langle n|\gamma_\mu|m\rangle\langle m|\gamma^\mu|n\rangle \times \left[\frac{1}{2}\omega_{nm} + \frac{\omega_{nm}}{e^{2\pi\omega_{nm}} - 1}\right] , \quad (26)$$

where we have summed a convergent geometric series

$$\sum_{p=1}^{\infty} e^{2\pi p\omega_{nm}} = \frac{1}{e^{2\pi\omega_{nm}} - 1} ,$$

which arises as the sum of the residues contributed from the infinitude of poles enclosed by the contour and located at $\xi = -2\pi i p$ for $p = 1, 2, 3, \ldots$. (Recall that if n is the ground state, then $\omega_{nm} < 0$.) This is our primary result, reinserting the constants into the expression in parentheses on the last line of (26) yields for the contents of these parentheses

$$\frac{1}{2}\hbar\omega_{nm} + \frac{\hbar\omega_{nm}}{e^{2\pi c\omega_{mn}/a} - 1} , \quad (27)$$

which we see is the Planck blackbody spectral distribution, complete with the so-called zero-point term. However, as we saw in Sec. III, the $\frac{1}{2}\hbar\omega_{nm}$ does not correspond to a vacuum spectrum but rather to the self-field spectrum of an inertially moving detector. When we boost into an accelerating frame we get back the inertial term plus the Planckian term at a temperature of

$$T_a = \frac{\hbar a}{2\pi k c} . \tag{28}$$

So then it appears as if the accelerating detector is exposed to a thermal bath of photons at temperature T_a; just as it appears as if it is also being exposed to a zero-point field embodied in the $\frac{1}{2}\hbar\omega$. Neither set of photons are physically real. Since any nearby inertially moving detector would detect no photons, the accelerating detector cannot be detecting real photons either. By the equivalence principle we can conclude the same thing about a detector placed in a uniform gravitational field of strength a. The field does not create a bath of photons, rather the detector is responding directly to the local curvature of space-time. The energy required to excite the detector into a thermal superposition of states is tapped directly from the metric without the intermediary of any electromagnetic radiation.

If our results generalize to the case of black holes, then we would conclude that although a black hole has the capability of directly exciting detectors in its neighborhood, it does not necessarily do so by emitting a flood of thermal radiation. If this is indeed the case, then black holes do not radiate in the ordinary sense of that word, i.e. they do not lose mass or energy via this mechanism unless a detector is actually present.

V. CONCLUSION

We have shown that the Unruh effect can be calculated within the context of a source-field theory; we conclude that the thermal response of the detector arises not through an interaction with real photons in the surrounding space, but from the spectrum of its self-field which has become altered by the change to a noninertial frame. This indicates that the detector is becoming excited directly by changes in the metric tensor. If all such responses of the detector can be attributed to modifications of the self-field, as opposed to modifications in a vacuum field, it would seem unlikely then that black holes are emitting real, physical radiation.

Davies has argued that, in particular, the concept of the photon is not well defined in curved space-time quantum electrodynamics, since in a curved space-time different detectors respond differently to the vacuum field and "detect" different photon spectra.[3] It is our contention that it is the vacuum field in QED that is not well defined; if it were identically zero to begin with it would not cause trouble in any space-time, curved or otherwise. Davies has persistently pointed out that any discussion of the vacuum field must always concern itself with the worldline of the detector which registers departures from that vacuum. Herein lies the key. Since in curved space-time the concept of vacuum and detector cannot be either conceptually or operationally separated, this is a clue that they are really two sides of the same coin. The fact that the source field and the vacuum field are closely related is well documented;[10] but in the framework of standard QED it appears as if both are always required for the internal consistency of the theory. The fact that the vacuum field is required in standard QED is a direct consequence of the second quantization procedure. In the present approach there is no quantization of the EM field, and yet we obtain correct results at least to order α, for radiative effects thought to require second quantization for their explanation.[6,8] So the question is as follows: Can we always get the correct results without recourse to some sort of vacuum fluctuation? If we can set the zero-point field identically equal to zero for all moments in the Wightman, two-point correlation function, then the concept of photon might be rescued. There are no photons in the Minkowski vacuum or any other vacuum to be counted by any detector—regardless of its state of motion or history.

The self-field approach to quantum electrodynamics as presented in this work has been used successfully to first order in α to account for nonrelativistic and relativistic formulas for spontaneous emission, the Lamb shift, and $g-2$, all in free space.[6,8] This approach has also been used to compute various apparatus dependent contributions to these effects; again to lowest order. By enlarging the notion of boundary to include the effects of a non-Minkowskian event horizon we have, in the present work, accounted for the Unruh effect. Work is in progress to show that the Unruh effect is essentially a classical phenomenon, and to analyze the response of an atom to Planck blackbody radiation in a more general setting from the point of view of self-fields. (Since spontaneous emission and the Lamb shift have the classical limits of line broadening and level shift, the Unruh effect should also have such a classical analog.) The extension of the self-field approach to higher orders of the fine-structure constant has until now been hampered by delicate and complicate calculations of the wave functions needed in the general nth order iteration of the self-field contribution to the total action. Some progress is now being made in this direction.

---

*Present address: Max-Planck-Institute for Quantum Optics, D-8046 Garching, Federal Republic of Germany.

[1]S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
[2]W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
[3]N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1982), pp. 48–58.
[4]T. H. Boyer, Phys. Rev. D **21**, 2137 (1980); **29**, 1089 (1984).
[5]J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975), pp. 798–801.
[6]A. O. Barut and J. Kraus, Found. Phys. **13**, 189 (1983); A. O. Barut and J. F. van Huele, Phys. Rev. A **32**, 3187 (1985); A.

O. Barut and N. Unal, Physica **142A**, 467 (1987); **142A**, 488 (1987); A. O. Barut and Y. I. Salamin, Phys. Rev. A **37**, 2284 (1988); A. O. Barut, Phys. Scr. **T21**, 18 (1988).

[7]A. Sandage, Astrophys. J. **133**, 355 (1961); A. Sandage, Observatory **88**, 91 (1968).

[8]A. O. Barut and J. P. Dowling, Phys. Rev. A **36**, 649 (1987); **36**, 2550 (1987); **39**, 2796 (1989); A. O. Barut and J. P. Dowling, Z. Naturforsch. **44a**, 1051 (1989); A. O. Barut, J. P. Dowling, and J. F. van Huele, Phys. Rev. A **38**, 4405 (1988).

[9]E. T. Jaynes, *Coherence and Quantum Optics*, edited by L. Mandel and E. Wolf (Plenum, New York, 1973), p. 35.

[10]P. W. Milonni, Phys. Scr. **T21**, 102 (1988).

Self-field quantum electrodynamics: The two-level atom

A. O. Barut
Department of Physics, University of Colorado, Campus Box 390, Boulder, Colorado 80309

Jonathan P. Dowling
Max-Planck-Institute for Quantum Optics, Postfach 1513, Garching D-8046, Federal Republic of Germany
(Received 26 October 1989)

We use a self-field approach to quantum electrodynamics (QED) to show how one may obtain spontaneous emission and the Lamb shift in a two-level atom without second quantization of the radiation field. In addition, we compare the self-field formalism to that of the neoclassical theory of electrodynamics advanced by Crisp and Jaynes [Phys. Rev. **179**, 1253 (1969)]. We show that the neoclassical model can be obtained from the self-field approach used here, but that the two are not equivalent. In particular, the self-field approach appears to give a more complete description of radiative processes. Finally, we show that the neoclassical theory's prediction of a nonexponential "chirruped" decay is most likely a mathematical artifact of the improper application of the superposition principle in a nonlinear model where such a principle does not hold. A correct treatment with self-field QED yields the usual exponential decay dynamics.

I. INTRODUCTION

In classical electrodynamics one realizes that the Lorentz equation of motion for a charge in an electromagnetic (EM) field is incomplete, inasmuch as it does not include radiation reaction. Considerations such as this have led to the Abraham-Dirac-Lorentz (ADL) equation of motion which, in covariant form, can be written as[1]

$$m\ddot{z} = F^{\text{ext}}\dot{z} + \frac{2\alpha}{3}(\dddot{z} + \dot{z}\ddot{z}^2) , \quad (1)$$

where $z = z_\mu$ is the coordinate of the charge $q = e$, the dots denote differentiation with respect to the propertime τ, and $\alpha = e^2/4\pi$. (We use throughout this paper the convention $c = \hbar = 1$.) A covariant external force $F = F_\mu$ is allowed for also. In order to arrive at this equation (1), it is necessary that the entire problem be treated covariantly from the outset, with nonrelativistic approximations possible after the formula given above has been specified.

A very interesting derivation of Eq. (1) was given by Wheeler and Feynman using their action at a distance formulation of classical electrodynamics.[2] The idea goes back to a paper by Tetrode,[3], which shows that all of classical electrodynamics—Maxwell's equations and the ADL equation of motion—can be derived from a single unified action principle, if one demands that an accelerating charge produces a field which is symmetric in the retarded and advanced solutions to Maxwell's equations. In such a theory, the contributions of radiation reaction to the Lorentz equation of motion arise very naturally. It is well known that Wheeler and Feynman never produced a quantum version of this theory, although Süssman has presented a second quantized version.[4] The self-field approach to quantum electrodynamics (QED), as proposed by Barut and his co-workers, falls in between these two extremes. We replace the classical particle trajectory z_μ with either the scalar Schrödinger wave function ψ, the Pauli two component spinor ϕ, or finally the Dirac four-component spinor Ψ. With the Dirac spinor version the theory is fully covariant and may now be studied as a candidate for a complete theory of QED. At no point do we second quantize either the matter or the radiation field. Süssman, who does second quantize the fields in his quantum version of the Wheeler-Feynman approach, arrives at a correct explanation of spontaneous emission with the right Einstein A coefficient. This, we shall see, is also possible even at the atomic level where the particle is treated nonrelativistically as being described by a Schrödinger wave function, and nothing is second quantized. This result is understandable if we think of spontaneous emission as the quantum analog of the classical radiation reaction line broadening of an oscillating charge. Since the Wheeler-Feynman action accounts for radiation reaction naturally, spontaneous emission is a logical consequence of this approach, even if the EM field is not quantized. Notice that in our method there can be no EM vacuum field fluctuations since the field is not quantized. This precludes the notion of zero-point fluctuations as the physical cause of spontaneous emission in our picture.[5] If a semiclassical theory is defined as a theory which is not second quantized, then self-field QED has been quite a successful semiclassical theory (at least to order α) in accounting for quite an array of phenomena thought to require at least the second quantization of the radiation field for their explanation. Both relativistic and nonrelativistic accounts of spontaneous emission, the Lamb shift, and $g-2$ have been given.[6] Nonrelativistic calculations of cavity-induced changes to these effects have been carried out also, as well as a calculation of the Unruh effect (whereby an accelerating detector senses a bath of thermal radiation).[7] In the present paper we show how the theory can be used to treat the decay dynamics of a two-level atom, and that the self-field theory in some sense contains—but is not equivalent to—the neoclassical theory of Crisp and Jaynes.[8]

The paper is organized as follows. First, we present a review of self-field QED, emphasizing the point that the theory is a quantum generalization of the action at a distance approach to classical electrodynamics of Fokker, Schwarzshild, Tetrode, Wheeler, and Feynman. (Equivalently, it is a quantum theory of radiation reaction.) Second, we reduce the theory to that of a two-level atom and obtain the correct exponential spontaneous-emission decay law, as well as the Lamb shift contribution to the energy levels. Third, we indicate how in self-field QED we can arrive at the same erroneous decay law as that of the neoclassical theory if we assume that the superposition principle holds—which it does not in our nonlinear theory. Fourth, we review the neoclassical approach of Jaynes and show that it can lead to the same incorrect, chirruped exponential decay law. The conclusion is then that the same illegal use of superposition in the neoclassical theory—which is similarly nonlinear in the wave function ψ—leads to the wrong decay law. We will detail how the correct exponential decay can be recovered by avoiding recourse to the superposition principle in the neoclassical theory. We shall finally also indicate why we believe that the self-field approach to QED offers a more complete description of radiative corrections than does the neoclassical theory.

II. ACTION AT A DISTANCE ELECTRODYNAMICS

The action at a distance formulation of classical electrodynamics, as presented by Wheeler and Feynman,[2] presupposes an action principle used by Fokker, Schwarzschild and Tetrode.[1,3] Consider a number of charges e_i of mass m_i interacting by means of an action integral W, defined as

$$W=\sum_i \int d\tau\, m_i \dot{z}_i^2 + \sum_i \sum_j e_i e_j \int \int d\tau\, dv\, \dot{z}_i(\tau)\cdot\dot{z}_j(v) D(z_i - z_j) , \quad (2)$$

where τ and v are proper times, $z_i = z_i^\mu$ is the four-position of the ith particle, and integration is over all space-time. (We are using standard four-vector notation: $z \equiv z_\mu$, $z^2 \equiv z_\mu z^\mu = z\cdot z$, etc.) The $D(x-y)$ is an electromagnetic Green's function. In order for the variational problem to have a solution, the Green's function $D(x)$ must be symmetric under particle interchange $i \leftrightarrow j$ and also in past and future.[2] These requirements lead to the two equations

$$D=\tfrac{1}{2}(D^{\text{advanced}}+D^{\text{retarded}}) , \quad (3a)$$

$$D(z_i - z_j)=D(z_j - z_i) . \quad (3b)$$

We note that the usual Feynman propagator of QED satisfies both of these conditions. When we extend the theory to the quantum domain, the choice of a symmetric Feynman boundary condition of the form of (3a) will be *required* in order for the variational problem to have a solution. Hence the choice of such a propagator will not be *ad hoc,* but will arise as a natural requirement of the theory.[2,3] With certain further assumptions concerning boundary conditions, it is well known that the variation of the action (2) with respect to z_μ yields, for the Euler-Lagrange equations of motion, the ADL equation (1). Hence we have a classical action principle which yields radiation reaction. If one considers the classical motion of a harmonically bound charge with radiation reaction included, one finds that there arises a level shift and a line broadening to the energy of the oscillator.[1,9] We shall see that these classical phenomena have as their natural quantum analog the Lamb shift and spontaneous emission. From the self-field point of view, all quantum electrodynamic, radiation reaction effects are viewed as the quantum extensions of such classical effects. The problem now is to pose an action principle such as Eq. (2) in a quantum-mechanical setting. To see how to proceed, let us relate the action principle of (2) to one which resembles that of the usual classical field theory. The electromagnetic four-potential $A_\mu^{(i)}(x)$ of the ith particle at the point $x = x_\mu$ is given by

$$A_\mu(x)=e\int d\tau\, D(x-z(\tau))\dot{z}_\mu(\tau) , \quad (4)$$

where the subscript i has now been suppressed. Such a potential gives rise to a field tensor $F_{\mu\nu}$, which is symmetric in retarded and advanced fields; so long as the Green's function satisfies expression (3a).[1] The field tensor also obeys Maxwell's equations, provided we take the current density of the ith particle as

$$j_\mu(x)=e\int d\tau\, \dot{z}_\mu \delta(x-z(\tau)) . \quad (5)$$

If we insert expression (4) into expression (2) for the action W, we obtain

$$W=\sum_i \int d\tau\, m_i \dot{z}_i^2 + \sum_i \sum_j \int d\tau\, e_i \dot{z}_i(\tau) A_j(z_i(\tau)) , \quad (6)$$

which, by inspection of the current j of Eq. (5), we see is the usual classical field theoretical action with a $j\cdot A$ type interaction term. This procedure now give us a clue as to how to go about constructing quantum versions of the action principle embodied in expression (2).

(i) Write down the usual action for either the Schrödinger, Pauli, Or Dirac equation, with the $j\cdot A$ interaction.

(ii) Separate the EM potential according to $A_\mu = A_\mu^{\text{ext}} + A_\mu^{\text{self}}$, where A_μ^{ext} is some field arising from charges assumed to be at infinity and A_μ^{self} is the self-field of the charges in a localized interaction region.

(iii) Eliminate A_μ^{self} entirely from the total action by use of the Feynman Green's function $D(x)$ via the prescription

$$A_\mu^{\text{self}}(x)=e\int dx\, D_{\mu\nu}(x-y) j^\nu(y) , \quad (7)$$

which is a generalization of Eq. (4).

Here the j_μ are quantum electron density currents appropriate to the wave functions ψ, ϕ, or Ψ. When these steps are carried out, one is left with an action principle in which the self-field potential A_μ^{self} has been entirely eliminated. The external field $A_\mu^{\rm ext}$ still remains; however, it too could be eliminated if we took our interaction region to be the entire universe. In this case we would have a pure, quantum, action at a distance theory; in particular, it would be a generalization of the Wheeler-Feynman approach contained in Eq. (2). We would hope that the quantum versions of the theory now account for radiation reaction automatically, just as the classical version does so. We shall see, furthermore, that the classical radiation effects of line broadening and level shift have, as their expression in the quantum versions of the theory, spontaneous emission and the Lamb shift. This might be expected from correspondence principle grounds. We now formalize these motivational comments and remarks into a presentation of the self-field theory of quantum electrodynamics.

III. SELF-FIELD QUANTUM ELECTRODYNAMICS

Maxwell's equations and the quantum-mechanical (QM) equations of motion—including radiative or radiation reaction effects—arise from a single action principle, if we use a Feynman Green's function to relate the electromagnetic potential to the current that produces it, via Eq. (7). It is postulated that there are no EM fields independent of the sources that produce them, and hence no possibility of vacuum field fluctuations. It is assumed that the field surrounding a charge can be split as $A_\mu = A_\mu^{\rm ext} + A_\mu^{\rm self}$, where the external field has as its source charges at infinity, while the self-field is the field produced by a localized charge in some interaction region. With this *ansatz* the nonhomogeneous Maxwell equation

$$F^{\mu\nu}{}_{,\mu} = ej^\nu \tag{8}$$

has the general solution

$$A_\mu(x) = A_\mu^{\rm ext} + e\int dy\, D_{\mu\nu}(x-y)j^\nu(y) , \tag{9}$$

where $D_{\mu\nu}(x-y)$ *must* be symmetric in retarded and advanced Green's functions. Its precise form will depend on the gauge, and also the overall boundary conditions on the EM field. The total action W may be written as the four-dimensional integral of an action density w,

$$W = \int dx\, w[x;\psi, A_\mu] . \tag{10}$$

The action density w will have the general form

$$w(x) = w_0(x) + eA_\mu j^\mu + \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} , \tag{11}$$

where the specific form of $w_0(x)$, the matter action, will depend on the extent to which a charge is treated nonclassically. If one uses integration by parts, the homogeneous Maxwell's equation, and the assumption that $A_\mu^{\rm self}$ is sufficiently localized, one obtains

$$\begin{aligned} w &= w_0 + eA_\mu^{\rm ext}j^\mu + \frac{e}{2}A_\mu^{\rm self}j^\mu \\ &= w_i + \frac{e}{2}A_\mu^{\rm self}j^\mu \\ &= w^{\rm ext} + w^{\rm self} . \end{aligned} \tag{12}$$

The interpretation is that $w^{\rm ext}$ is responsible for the usual electronic motion in an external field, while $w^{\rm self}$ contains radiation reaction effects or radiative corrections such as the Lamb shift and spontaneous emission, corresponding to level shifts and line broadening in the classical theory. With the definition (10), we find that the variation of W with respect to $A_\mu^{\rm self}$ yields, for the Euler Lagrange equations of motion,

$$\frac{\delta W}{\delta A_\nu^{\rm self}} - \partial_\mu \frac{\delta W}{\delta A_{\nu,\mu}^{\rm self}} = -ej^\nu + F^{\mu\nu}_{{\rm self},\mu} = 0 , \tag{13}$$

which is the inhomogeneous Maxwell equation, provided we have identified

$$\frac{\delta W}{\delta A_\mu^{\rm self}} = -ej^\mu . \tag{14}$$

This development has thus far been independent of the choice of the action density w_0. We now summarize the action densities and their corresponding currents for the most important cases.

(i) *Classical action density and current*:

$$w_i = m\dot z^2 - eA_\mu \dot z^\mu , \tag{15a}$$

$$j^\mu = \int d\tau\, e\dot z^\mu \delta(x - z(\tau)) . \tag{15b}$$

(ii) *Schrö*dinger action density and current:

$$w_i = \psi^* \left[\frac{1}{2m}(\overleftarrow{\nabla} + ie\,\mathbf{A})\cdot(\overrightarrow{\nabla} - ie\,\mathbf{A}) + eA_0 - i\frac{\partial}{\partial t} \right] \psi , \tag{16a}$$

$$j^\mu = \psi^* \left[1, \frac{1}{2mi}\overleftrightarrow{\nabla} - \frac{e}{m}\mathbf{A} \right] \psi . \tag{16b}$$

(iii) *Pauli action density and current*:

$$w_i = \phi^* \left[\frac{1}{2m}[(\overleftarrow{\nabla} + ie\,\mathbf{A})\cdot\sigma] \times [\sigma\cdot(\overrightarrow{\nabla} - ie\,\mathbf{A})] + eA_0 - i\frac{\partial}{\partial t} \right] \phi , \tag{17a}$$

$$j^\mu = \phi^* \left[1, \frac{1}{2mi}\overleftrightarrow{\nabla} + \frac{1}{2m}(\overleftarrow{\nabla}\times\sigma - \sigma\times\overrightarrow{\nabla}) - \frac{e}{m}\mathbf{A} \right] \phi . \tag{17b}$$

(iv) *Dirac action density and current*:

$$w_i = \overline{\Psi}[\gamma^\mu(i\partial_\mu - eA_\mu) - m]\Psi , \tag{18a}$$

$$j^\mu = \overline{\Psi}\gamma^\mu\Psi . \tag{18b}$$

Variation of W with respect to z_μ, ψ, ϕ, or Ψ yields, respectively, the ADL, Schrödinger, Pauli, or Dirac equations of motion. It must be emphasized that the A_μ which appear in the equations above are not just the external field A_μ^{ext} alone, but rather the sum of $A_\mu^{\text{ext}}+A_\mu^{\text{self}}$, as given in Eq. (9). So, unlike the usual semiclassical theory, the action densities listed above contain nonlinear and nonlocal terms of the general form

$$W^{\text{self}}=\frac{e^2}{2}\int\int dx\, dy\, j^\mu(x)D_{\mu\nu}(x-y)j^\nu(y)\ , \tag{19}$$

which are responsible for radiative corrections. [Note the similarity between this expression and the Wheeler-Feynman double integral of Eq. (2) for the classical action, in the self-interaction case where $i=j$.] In particular, we should notice that because the equations of motion are nonlinear *the superposition principle does not hold*. It is not possible to expand the exact solutions of the exact nonlinear equation as a superposition of solutions to the approximate, linear equation which does not contain W^{self}. In addition, the terms of the form (19), which now appear in the action integral W, are not perturbations which can be turned on or off at will. They are an integral part of the entire action and their inclusion is always required in order to have a *complete* equation of motion which includes radiation reaction, in analogy to the classical ADL equation (1). For example, if $A_\mu^{\text{ext}}=(-Z_e/r,0)$, the static Coulomb potential, then the usual hydrogenic wave functions ψ_{nlm}^0 with eigenvalues E_n^0 are not—even in principle— solutions to the complete Schrödinger action, which contains now the nonlinear term W^{self} given in Eq. (19). The conclusion is that the hydrogen atom has no precisely defined sharp energy levels, other than the ground state.[5-7] The excited states cannot be stable, according to the self-field picture, due to radiation reaction. Hence they are never precise levels—but they always have a nonzero linewidth which manifests itself as spontaneous emission. Mathematically, the ψ_{nlm}^0 form a complete set of states, and in the usual perturbation theory the solution to the perturbed eigenvalue problem can be expanded as a linear superposition of this complete set. Such an approach would not be correct here, since the principle of linear superposition does not hold for our equations of motion—they contain nonlinear current interaction terms of the form (19). One must be wary of blindly applying the machinery of QM to a problem without regard for the hypotheses upon which such an application is based. (We should point out that even in the standard approach to QED the radiative corrections are not really perturbations either. If one adheres to the notion that radiative effects have their origin in the vacuum field fluctuations, then such corrections form a necessary part of the problem, since the vacuum fluctuations can not be turned off—even in principle. Hence in standard QED the hydrogen atom cannot have exact eigenstates. All of the states, except for the ground state, will have a spread to them which cannot, under any circumstances, be eliminated.)

IV. SELF-FIELD QED FOR A TWO-LEVEL ATOM

We shall now derive the spontaneous-emission rate and Lamb shift for a two-level atom, and attribute them to be physical consequences of the covariant inclusion of the electron's self-field. The interpretation is that spontaneous emission and the Lamb shift are triggered by the electron's radiation reaction field—in complete analogy to the classical account of the line broadening and level shift of the energy of a harmonically bound charge.

It is sufficient for a two-level atom to consider a Schrödinger action principle. The total Schrödinger action can be obtained by inserting the expressions (16) into the action density (12) and then by integrating over all of space-time as per the definition given by Eq. (10). The result is

$$W=\int dx\,\psi^*(x)\left[-\frac{1}{2m}\nabla^2-i\frac{\partial}{\partial t}+\frac{ie}{m}\mathbf{A}^{\text{ext}}\cdot\nabla+\frac{ie}{2m}\mathbf{A}^{\text{self}}\cdot\nabla+\frac{e^2}{2m}(\mathbf{A}^{\text{ext}})^2+\frac{e^2}{2m}\mathbf{A}^{\text{ext}}\cdot\mathbf{A}^{\text{self}}\right.$$
$$\left.+eA_0^{\text{ext}}+\frac{e}{2}A_0^{\text{self}}+\frac{ie}{2m}\nabla\cdot\mathbf{A}^{\text{ext}}+\frac{ie}{4m}\nabla\cdot\mathbf{A}^{\text{self}}\right]\psi(x)\ , \tag{20}$$

where $dx\equiv d^4x$. If we were to take $A_\mu^{\text{self}}=0$ in this expression, we would recover precisely what is usually called semiclassical electrodynamics. However, we can not set the self-field to zero and maintain a complete theory of electronic motion which includes radiative effects. This was first pointed out by Schrödinger.[10] Even formally A_μ^{self} can never be zero, for it is always given by Eq. (9), which makes it proportional to the current. The only way A_μ^{self} can be zero at all points in space-time is if the electron four-current is zero at all points in space-time—in which case we have no electron.

Variation of (20) with respect to ψ^* will yield the usual Schrödinger equation, augmented by new, nonlinear terms which contain A_μ^{self}. Using a few techniques from S-matrix theory, we can work directly with the total action (20) and extract radiative corrections to the usual Coulomb energies. Notice that A_μ^{self} depends on j_μ, via Eq. (9), but that j_μ also depends on A_μ^{self} via equation (16b). Hence we have a "feedback loop" in the equations, with each cycle of the loop contributing corrections of successively higher orders in the fine-structure constant α. (In this work we keep only corrections to first order in α.) To this order the $\mathbf{A}^{\text{ext}}\cdot\mathbf{A}^{\text{self}}$ term of Eq. (20) is negligible. For weak external fields $(\mathbf{A}^{\text{ext}})^2$ is also negligible.

In fact, since we are interested in hydrogenic atoms, we may set $\mathbf{A}^{\text{ext}}=0$ and $A_0^{\text{ext}}=-Ze/r$. Finally, a choice of the Coulomb or radiation gauge will eliminate both $\nabla\cdot\mathbf{A}^{\text{ext}}$ and $\nabla\cdot\mathbf{A}^{\text{self}}$. With these observations the action W can be written

$$W=\int dx\,\psi^*(H_0+H_1+H_2)\psi$$
$$=W_0+W_1+W_2\ , \tag{21}$$

where

$$H_0=-\frac{1}{2m}\nabla^2+eA_0^{\text{ext}}-i\frac{\partial}{\partial t}\ , \tag{22a}$$

$$H_1=\frac{ie}{2m}\mathbf{A}^{\text{self}}\cdot\nabla\ , \tag{22b}$$

$$H_2=\frac{e}{2}A_0^{\text{self}}\ . \tag{22c}$$

It turns out that H_0 is responsible for the usual Coulombic motion, H_1 gives rise to spontaneous emission and the Lamb shift, and finally, H_2 corresponds to a mass renormalization analogous to that which appears in the classical theory of radiation reaction.[1,5]

As we mentioned above, we are using the Coulomb gauge $\nabla\cdot\mathbf{A}=0$. In this gauge the components of the Green's function $D_{\mu\nu}(x-y)$ become

$$D_{ij}(x-y)=\frac{1}{(2\pi)^4}\int dk\frac{e^{-ik(x-y)}}{k^2+i\epsilon}(\delta_{ij}+\hat{k}_i\hat{k}_j)\ , \tag{23a}$$

$$D_{00}(x-y)=\frac{1}{(2\pi)^4}\int dk\frac{e^{-ik(x-y)}}{\lambda^2+i\epsilon}\ , \tag{23b}$$

$$D_{i0}(x-y)=D_{0i}(x-y)=0\ , \tag{23c}$$

where $\lambda\equiv|\mathbf{k}|^2$, $k^2\equiv k^\mu k_\mu$, and the $+i\epsilon$ in the denominator insures that the correct symmetry between retarded and advanced solutions to Maxwell's equations is obtained. With this choice of Green's function the equation (9) for the self-field can be written as

$$\mathbf{A}^{\text{self}}=-\frac{e}{(2\pi)^4}\int\int dy\,dk\frac{e^{-ik(x-y)}}{k^2+i\epsilon}\times[\mathbf{j}(y)-\hat{\mathbf{k}}(\hat{\mathbf{k}}\cdot\mathbf{j}(y))]\ , \tag{24a}$$

$$A_0^{\text{self}}(x)=\frac{e}{(2\pi)^4}\int\int dy\,dk\frac{e^{-ik(x-y)}}{\lambda^2+i\epsilon}\rho(y)\ , \tag{24b}$$

where ρ and $\mathbf{j}$ are the time and space components of the current j_μ as given in Eq. (16b). In our notation above we use $dy\equiv d^4y$, $dk\equiv d^4k$, and $\hat{\mathbf{k}}\equiv\mathbf{k}/|\mathbf{k}|$.

If we now insert Eq. (16b) into the above expressions (24) and put these into the total action given in (21), we obtain

$$W_0=\int dx\,\psi^*(x)\left[-\frac{1}{2m}\nabla^2+eA_0^{\text{ext}}-i\frac{\partial}{\partial t}\right]\psi(x)\ , \tag{25a}$$

$$W_1=-\frac{\alpha}{(2\pi)^3m^2}\int\int\int dx\,dy\,dk\frac{e^{-ik(x-y)}}{k^2+i\epsilon}[\psi^*(x)\nabla_x\psi(x)]\cdot\{\psi^*(y)[\nabla_y-\hat{\mathbf{k}}(\hat{\mathbf{k}}\cdot\nabla_y)]\psi(y)\}\ , \tag{25b}$$

$$W_2=\frac{\alpha}{(2\pi)^3}\int\int\int dx\,dy\,dk\frac{e^{-ik(x-y)}}{\lambda^2+i\epsilon}\rho(x)\rho(y)\ , \tag{25c}$$

where $\alpha=e^2/4\pi$.

The ψ which appears in these equations (25) is assumed to be the function which minimizes the total action $W=W_0+W_1+W_2$. Equivalently, they are solutions to the augmented, integro-differential Schrödinger equation one obtains from the Euler Lagrange equations of motion when W is varied with respect to ψ^*. The point is that the $\psi(x)$ are as of yet unknown functions of the space-time coordinate $x=x_\mu=(t,\mathbf{x})$. Now, without the self-field contributions of W_1 and W_2, the action integral W_0 alone is minimized by the usual Coulombic wave functions, which we shall denote ψ_n^0 (where $n\equiv nlm$ contains all three hydrogenic quantum numbers). Physically, we should expect that there are true solutions ψ_n with energies E_n that minimize the complete nonlinear action W, and that these functions are in some sense close to the Coulombic functions ψ_n^0, with eigenenergies E_n^0, which minimize the linear action W_0 alone.

It is usual in perturbation theory to assume that since the Coulombic wave functions ψ_n^0 form a complete set, any solution ψ to the perturbed equation of motion can be expanded as a linear superposition given by

$$\psi(\mathbf{x},t)=\sum_n C_n(t)\psi_n^0(\mathbf{x})\ . \tag{26}$$

As a further *ansatz*, one can assume that the rapid oscillations proportional to $\exp(iE_n^0t)$ can be separated from the more slowly changing time behavior. Hence in atomic physics—especially in the two-level atom model—one writes expression (26) in the form

$$\psi(\mathbf{x},t)=\sum_n C_n(t)e^{iE_n^0t}\psi_n^0\ , \tag{27}$$

where the $C_n(t)$ are presupposed to vary only slowly in time when compared to the exponential factors. The E_n^0 are the eigenenergies of the ψ_n^0. Conservation of charge requires the normalization condition

$$\int d^3x\,|\psi(\mathbf{x},t)|^2=\sum_n|C_n(t)|^2=1\ . \tag{28}$$

This entire mathematical apparatus requires that the perturbation to the Schrödinger equation be linear, and thus that the principle of linear superposition holds. In self-field QED the nonlinear expression (19) is not a perturbation, but rather a required part of the equation of motion for a complete theory—it cannot be turned off. If one did try to treat this as a perturbation, using Eqs. (26) or (27), one could not trust the result because (19) is nonlinear in ψ and hence the superposition principle needed for expansions (26) or (27) does not hold. We shall see later in this paper that the erroneous consequence of making such an attack on the problem is the chirruped exponential decay predicted by neoclassical theory. Clearly a different approach is needed here.

In the eigenfunction expansion (26) we physically are making the *ansatz* that the spatial behavior is described by the known functions $\psi_n^0(\mathbf{x})$, and then we use this knowledge to obtain information about the time behavior of the unknown $C_n(t)$. We propose now to reverse this procedure by making instead the *Fourier expansion*[6]

$$\psi(\mathbf{x},t)=\sum_n \psi_n(\mathbf{x})e^{-iE_n t} , \tag{29}$$

in which we assume that the time behavior is known, and of the form $\exp(-iE_n t)$. It is now information about the unknown wave functions ψ_n and the corresponding energies E_n that we are looking for. Physically we would expect that the ψ_n and the energies E_n are approximately equal to the Coulombic wave functions ψ_n^0 with energies E_n^0. The ψ_n in addition would have a complex phase factor $\exp(i\phi_n)$, which would make the Fourier expansion (29) convergent. However, in a two-level atom, we can to first order of iteration replace ψ_n with ψ_n^0, since $n\in\{1,2\}$, and then solve for E_n, assuming that it has the form $E_n=E_n^0+\delta E_n$.

Let us now insert the Fourier expansion (29) for the action integral $W=W_0+W_1+W_2$ found in Eq. (25). For W_0 we find

$$\begin{aligned} W_0&=\int\int d^3x\,dt\sum_{n,m}\psi_n^*(\mathbf{x})H_0\psi_m(\mathbf{x})e^{i\omega_{nm}t}\\ &=\sum_{n,m}\int dt\langle n|H_0|m\rangle e^{i\omega_{n,m}t}\\ &=2\pi\sum_{n,m}\langle n|H_0|m\rangle\delta(\omega_{nm}) , \end{aligned} \tag{30}$$

where $\omega_{nm}\equiv E_n-E_m$.

If the ψ_n were Coulombic wave functions, this expression would be zero. The ψ_n^0 minimize W_0 alone. However, now the entire action W, of which W_0 is only one term, must be minimized as a whole. For our two-level atom discussion $n,m\in\{1,2\}$, but the general treatment holds for an atom with a complete set of levels.[5,6]

We now discuss the piece of the action W_1, which contains the Lamb shift and spontaneous emission. Inserting the expansion (29) into W_1 of (25b) we obtain, after carrying out the $x_0\equiv t$ and $y_0\equiv u$ time integrations,

$$W_1=-\frac{4\alpha}{3m^2}\sum_{n,m,p,q}\int d\lambda\,\lambda^2\frac{\delta(\omega_{nm}+\omega_{pq})}{\omega_{pq}^2-\lambda^2+i\epsilon}\times\langle n|\nabla|m\rangle\langle p|\nabla|q\rangle , \tag{31}$$

where we have used the dipole approximation (DA), i.e., $\exp[i(\mathbf{x}-\mathbf{y})]\approx 1$. The kets $|n\rangle$ are still exact solutions which we assume minimize the total action W. The δ function is satisfied by either of the two conditions

$$n=m,\quad p=q \tag{32a}$$

$$n=q,\quad m=p \tag{32b}$$

but one can show[6] that condition (32a) causes W_1 to vanish identically from parity considerations. This is because matrix elements of the form $\langle n|\nabla|n\rangle$ vanish for even azimuthal quantum numbers, and terms with opposite azimuthal quantum numbers will cancel in the summation in Eq. (31). Hence only the choice of (32b) gives any contribution to the total action. This then leaves us with the expression

$$W_1=\frac{4\alpha}{3}\sum_{n,m}\int d\lambda\,\lambda^2\frac{\omega_{nm}^2|\mathbf{x}_{nm}|^2}{\omega_{nm}^2-\lambda^2+i\epsilon} , \tag{33}$$

where we have used the relation $\langle n|\nabla|m\rangle\equiv\nabla_{nm}=-m_{\text{elec}}\omega_{nm}\mathbf{x}_{nm}$. Using the symmetry in the dummy indices n and m, we may write a partial fraction expansion

$$\frac{\lambda^2}{\omega_{nm}^2-\lambda^2}=\frac{\omega_{nm}}{\omega_{nm}-\lambda}-1 , \tag{34}$$

where the equality sign is understood to hold under the double summation $\sum_{n,m}$. The -1 in Eq. (34) corresponds to an energy shift proportional to ∇^2, and hence to a change in the electron mass. This term may be eliminated by renormalizing the electron mass,[5] leaving only the first term on the right-hand side of Eq. (34). The implied contour integration embodied in the $+i\epsilon$ in the denominator of (33) may be carried out by the usual prescription of writing the integrand as a principal part P plus a residue, as per

$$\frac{1}{\omega_{nm}-\lambda}=\mathrm{P}\left[\frac{1}{\omega_{nm}-\lambda}\right]-i\pi\delta(\omega_{nm}-\lambda) , \tag{35}$$

which gives

$$W_1=\frac{4\alpha}{3}\sum_{n,m}\omega_{nm}^3|\mathbf{x}_{nm}|^2\int_0^\infty\frac{d\lambda}{\omega_{nm}-\lambda}-\frac{4\pi\alpha}{3}i\sum_{\substack{n,m\\ m<n}}\omega_{nm}^3|\mathbf{x}_{nm}|^2 , \tag{36}$$

where the δ function of (35) contributes only if $m<n$. We now extract the contribution to level n alone, and convert to units of energy, via an S-matrix prescription.[11] For a bound-state problem, the total action W is related to the total invariant energy $\mathcal{E}$ of the system via

$$W_{fi}=2\pi\delta(E_f-E_i)\mathcal{E} , \tag{37}$$

and so the contribution from (36) to the total energy of level n is given by

$$\mathcal{E}_1^{(n)} = \frac{W_1^{(n)}}{2\pi} = \frac{2\alpha}{3\pi} \sum_m \omega_{nm}^3 |\mathbf{x}_{nm}|^2 \int_0^\infty \frac{d\lambda}{\omega_{nm} - \lambda} - \frac{2\alpha i}{3} \sum_{\substack{n,m \\ m<n}} \omega_{nm}^3 |\mathbf{x}_{nm}|^2 \equiv \delta E_n - i A_n \ . \quad (38)$$

The real part of the energy shift $\mathrm{Re}\{\mathcal{E}_1^{(n)}\}$ is the nonrelativistic result for the Lamb shift, first obtained by Bethe.[12] For a two-level atom the sum runs over $m=1,2$. If we define $\omega_{nm} = \omega_{21} \equiv \omega_0$ as usual, we have

$$A_1 = 0 \ , \quad (39a)$$

$$A_2 \equiv A = \frac{2\alpha}{3} \omega_0^3 |\mathbf{x}_{21}|^2 \ . \quad (39b)$$

This shows us that the ground state ψ_1 is stable, but that the excited state ψ_2 decays with a time characterized by $\tau \equiv 1/A$, where A is the usual Einstein coefficient of spontaneous emission. If we could prepare the two-level system in level two at time $t=0$, Eq. (29) would become

$$\psi(\mathbf{x},t) = \psi_2(\mathbf{x}) e^{-i(E_2^0 + \delta E_2)t - At} \ , \quad (40)$$

where a factor of 2 should now be included in the definition (39b) of A to account for the two polarization degrees of freedom of the photon. Equation (40) contains the usual exponential decay dynamics for a two-level atom, as found in standard QED. We now see that, self-field QED, if treated correctly, does not predict any nonstandard dynamics, such as the chirruped decay profile predicted by the neoclassical theory of Jaynes.

It can be shown[5] that the corrections to the total action coming from H_2 give rise to a static shift which is the same for all levels and hence unobservable, and also a small level shift contribution which, in the relativistic version of the theory, corresponds to the vacuum polarization term of Wichmann and Kroll. This effect is negligible in our two-level model, when compared to the dominant contribution arising from the real part of Eq. (38), and we will not consider it further in this paper.

V. SELF-FIELD QED ASSUMING SUPERPOSITION

Let us now see what would have happened had we used the usual, but inadmissable, expansion (27), together with the conservation of electronic charge condition (28), all *instead* of the Fourier expansion (29). [Recall that an expansion such as (27) in terms of a superposition of known eigenstates is not valid because the solutions of the complete nonlinear Schrödinger equation are not known—and because there is no superposition principle for a nonlinear equation.] For a two-level atom the expansion (27) and condition (28) become

$$\psi(\mathbf{x},t) = C_1(t)\psi_1^0(\mathbf{x}) e^{-iE_1^0 t} + C_2(t)\psi_2^0(\mathbf{x}) e^{-iE_2^0 t} \ , \quad (41a)$$

$$|C_1(t)|^2 + |C_2(t)|^2 = 1 \ . \quad (41b)$$

Following the usual development of the two-level atom model, we assume a Hamiltonian of the form

$$H = H_0 + H' \ , \quad (42)$$

where

$$H_0 \psi_n^0 = E_n^0 \quad (n=1,2) \quad (43)$$

and

$$\rho = \langle \psi | \psi \rangle = |C_1|^2 + |C_2|^2 = 1 \ , \quad (44)$$

with

$$H'_{nm} \equiv \langle n | H' | m \rangle \ . \quad (45)$$

We define as before $\omega_0 \equiv E_2 - E_1$. The equations of motion become, in all generality for two levels,[13]

$$i\dot{C}_1 = C_1 H'_{11} + C_2 H'_{12} e^{-i\omega_0 t} \equiv M_{11} + M_{12} \ , \quad (46a)$$

$$i\dot{C}_2 = C_1 H'_{21} e^{i\omega_0 t} + C_2 H'_{22} \equiv M_{21} + H_{22} \ . \quad (46b)$$

So far we have done nothing but summarize the theory of a dynamic two-level atom with a perturbation. In free space, however, the only possible candidate for a perturbation is the radiation reaction response of the atom to the electronic self-field. At a simple level it can be showed that the results of self-field QED arise as a self-induced Stark and Zeeman effect which arises when the electron cloud responds to its own electric field and magnetic fields. The Stark level shifts then are those of the usual Lamb shift.

We now take $H' \equiv H_1 + H_2$ as our perturbation, where H_1 and H_2 are given in Eq. (22). (The self-field contribution from H' are not perturbations, we recall, and already at this stage we should not expect this procedure to yield correct results.) From parity considerations it is easy to show that[13]

$$H_{11}^{(2)} = H_{22}^{(1)} = 0 \ , \quad (47a)$$

$$H_{12}^{(1)} = H_{21}^{(1)*} \ , \quad (47b)$$

$$H_{12}^{(2)} = H_{21}^{(2)} = 0 \ . \quad (47c)$$

To calculate the nonzero matrix elements, we begin with H_1 in Eq. (22). Into this expression we insert the EM vector potential $\mathbf{A}^{\mathrm{self}}$ from (24a) with the current $\mathbf{j}$ taken from (16b). This yields the result

$$H_{12}^{(1)} = -\frac{\alpha}{3m^2\pi^2}\int\int\int d\lambda\, du\, d\omega \frac{\lambda^2 e^{-i\omega t}}{\omega^2-\lambda^2+i\epsilon}\nabla_{12}\cdot[C_2^*(u)C_1(u)\nabla_{21}e^{i(\omega+\omega_0)u}+C_1^*(u)C_2(u)\nabla_{12}e^{i(\omega-\omega_0)u}] , \quad (48)$$

where u is a dummy time integration variable. (We have used the dipole approximation.) Since the $C_i(t)$ are assumed to vary only slowly with time, we may replace $C_i(u)\approx C_i(t)$ in the integrand, allowing us to carry out the u integration. In the rotating-wave approximation we neglect the terms of expression (48), which contain the exponential $\exp[i(\omega+\omega_0)]$ and then the matrix elements $M_{ij}^{(1)}$ of Eq. (46), arising from the perturbation H_1, become

$$M_{12}^{(1)} = -\frac{2\alpha}{3\pi}\omega_0^2|\mathbf{x}_{21}|^2 I(\omega_0)|C_2|^2 C_1 , \quad (49a)$$

$$M_{21}^{(1)} = -\frac{2\alpha}{3\pi}\omega_0^2|\mathbf{x}_{21}|^2 I^*(\omega_0)|C_1|^2 C_2 , \quad (49b)$$

where we have defined

$$I(\omega_0)\equiv\int d\lambda\frac{\lambda^2}{\omega_0^2-\lambda^2+i\epsilon} , \quad (50)$$

which is identical to the integral which appears in W_1. Hence we expect $I(\omega_0)$ to contribute a level shift and a line broadening as before. Similarly, as in Eq. (34), in $I(\omega_0)$ we can renormalize away a linearly divergent mass term as Bethe does,[12] leaving a logarithmically divergent contribution to the Lamb shift, and a complex residue which mediates the decay of the atom, i.e.,

$$I(\omega_0)\to\omega_0\ln\left[\frac{\Lambda}{\omega_0}\right]-i\pi\omega_0 , \quad (51)$$

where the cutoff Λ is usually taken as $\Lambda=m$.

We now calculate the matrix elements $M_{ij}^{(2)}$, which come from the perturbation H_2. Using the definition of H_2 from Eq. (22c), the charge density $j^0\equiv\rho$ from (16b), and the charge conservation condition (44); we obtain in the dipole approximation

$$H_{11}^{(2)} = H_{22}^{(2)} = \frac{\alpha}{\pi}\Lambda , \quad (52)$$

where Λ is the same photon cutoff parameter used in (51). This divergent energy shift is level independent, and hence unobservable. (It is the same for all levels, and hence can be subtracted off by rescaling the energy axis. This divergence is an artifact of the dipole approximation.) Combining the results of (49) and (52), we have for the time evolution equations (46)

$$i\dot{C}_1 = \frac{\alpha\Lambda}{\pi}C_1 - \frac{2\alpha}{3\pi}\omega_0^3|\mathbf{x}_{21}|^2 \times\left[-\ln\left[\frac{\Lambda}{\omega_0}\right]-i\pi\omega_0\right]|C_2|^2 C_1 , \quad (53a)$$

$$i\dot{C}_2 = \frac{\alpha\Lambda}{\pi}C_2 - \frac{2\alpha}{3\pi}\omega_0^3|\mathbf{x}_{21}|^2 \times\left[-\ln\left[\frac{\Lambda}{\omega_0}\right]+i\pi\omega_0\right]|C_1|^2 C_2 . \quad (53b)$$

Now take Eq. (53a) and multiply it by C_1^*; then take the complex conjugate of (53a) and multiply through by C_1. Add the resultant equations together, and make use of the charge conservation condition (44). The final equations that remain after these operations are

$$\frac{d}{dt}|C_1|^2 = 2A(1-|C_1|^2)|C_1|^2 , \quad (54a)$$

$$\frac{d}{dt}|C_2|^2 = -\frac{d}{dt}|C_1|^2 , \quad (54b)$$

where A is the usual Einstein A coefficient for spontaneous emission, defined as

$$A\equiv\frac{2\alpha}{3}\omega_0^3|\mathbf{x}_{21}|^2 . \quad (55)$$

Notice how the terms containing the cutoff Λ have dropped out. These terms correspond to level shifts, and do not affect the dynamics of the spontaneous decay of level two into level one. If we define $X\equiv|C_1|^2$ and $Y\equiv|C_2|^2$, Eqs. (54) may be integrated to give

$$X(t) = \frac{1}{Ke^{-2At}+1} , \quad (56a)$$

$$Y(t) = \frac{1}{Le^{+2At}+1} , \quad (56b)$$

which are the chirruped hyperbolic decay profiles predicted by the neoclassical theory of Jaynes. (The K and L are constants of integration, with $K=1/L$.)

Now, how is it possible that we have lost the purely exponential decay profile of Eq. (40)? The physical input to the theory has not changed—only the mathematical analysis leading to the final result given above in Eq. (56). Although the decay constant $\tau=1/A$ is correct, the functional form is not. It is our contention that the chirruped decay arises not from incorrect physics, but rather from the incorrect use of the superposition principle, as it appears in the form of Eq. (41a). The perturbation H' is nonlinear in ψ and hence the total wave function that solves the perturbed, nonlinear, equation of motion is not necessarily expandable as a linear combination of ψ_1 and ψ_2. Such a procedure can yield at most an approximately correct solution. And indeed we see that the solutions (56) decay with the correct time constant A, and exhibit the correct exponential decay asymptotically as $t\to\infty$. The decay, however, is not correct for short times.

VI. REVIEW OF NEOCLASSICAL ELECTRODYNAMICS

The neoclassical theory of Crisp and Jaynes[8] is essentially equivalent to an idea of Schrödinger[10] and Fermi:[14] to include some quantum analog of classical radiation reaction effects in the Schrödinger equation to account for spontaneous emission. In this sense the neoclassical theory is in the same spirit as self-field QED. Fermi's development is essentially the same as that of Crisp and Jaynes, and so we present primarily his methodology here. We will restrict ourselves to a two-level atom discussion, to maintain consistency with the previous presentation.

Consider a solution $\psi(\mathbf{x},t)$ to Schrödinger's equation

$$i\frac{\partial\psi}{\partial t}=H\psi \ , \tag{57a}$$

$$H\equiv H_0+H' \ , \tag{57b}$$

$$H_0\equiv-\frac{1}{2m}\nabla^2+V_0(\mathbf{x}) \ , \tag{57c}$$

$$H'\equiv V'(\mathbf{x},t) \ , \tag{57d}$$

where V' is supposed to correspond to a radiation reaction potential of some sort. We suppose that the electron charge density is given as usual by $e\psi^*\psi$, and conservation of charge requires that $\int d^3x\rho=1$. The electric dipole moment can be written as

$$\mathbf{p}=\int d^3x\,\mathbf{x}\rho(\mathbf{x}) \ , \tag{58}$$

which is equal to the classical expression. The solutions $\psi_n^0(\mathbf{x},t)$ to the unperturbed equation $H=H_0$ can be written

$$\psi_n^0(\mathbf{x},t)=\psi_n^0(\mathbf{x})e^{-iE_n^0t} \quad (n=1,2) \tag{59}$$

where the $\psi_n^0(\mathbf{x})$ are solutions to the stationary equation $H_0\psi_n^0=E_n^0\psi_n^0$, and are normalized as usual as $\langle n|m\rangle=\delta_{nm}$.

In order to get the neoclassical theory of spontaneous emission we make the following *ansatz* for the form of the potential $V'(\mathbf{x},t)$:

$$V'(\mathbf{x},t)=\frac{2}{3}\left[\frac{e}{2\pi}\right]\mathbf{x}\cdot\dddot{P} \ , \tag{60}$$

which is taken directly from classical electrodynamics.[1] Expression (60) does not arise naturally in the theory of neoclassical EM, but is inserted in a rather *ad hoc* fashion in order to *make* the neoclassical theory. Notice that with the inclusion of (60) in the Hamiltonian, the resultant Schrödinger equation is nonlinear and nonlocal since V' depends on the three-dimensional space integral of $\mathbf{x}\psi^*\psi$. The situation is very similar to what was obtained in the self-field theory in Eq. (21), except that the neoclassical Schrödinger equation is not as complete. In addition to being a rather arbitrary prescription, as it stands it can account only for spontaneous emission and not the Lamb shift, vacuum polarization, $g-2$, etc.

To continue with the analysis in the context of neoclassical theory, one next assumes that the full solution of the perturbed equation (57a) can be expanded as a *linear superposition* of the eigenstates of the unperturbed equation. As we saw previously in the context of self-field QED, such an assumption is not correct due to the nonlinearity inherent in the perturbed Schrödinger equation (57a). This, we believe, is the erroneous step in neoclassical theory which gives the unphysical chirruped decay profile. To see that this is indeed so, we assume a linear superposition such as in Eq. (41). With this expansion, the potential V' becomes

$$V'(\mathbf{x},t)=\frac{2i\alpha}{3}\sum_{n,m}(\mathbf{x}\cdot\mathbf{x}_{nm})\omega_{nm}^3e^{i\omega_{nm}t}C_n^*C_m \ , \tag{61}$$

where $\omega_{nm}\equiv E_n^0-E_m^0$ and $\mathbf{x}_{nm}=\langle n|\mathbf{x}|m\rangle$, as before. If we insert Eq. (61) into (57), along with expansion (41), and then operate on the result expression with $\int d^3x\,\psi_k^*(\mathbf{x})$ and sum both sides, we arrive at the following time evolution equation:

$$\dot{C}_k=-\frac{2\alpha}{3}\sum_{l,n,m}C_lC_n^*C_m\omega_{nm}^3(\mathbf{x}_{kl}\cdot\mathbf{x}_{nm})e^{i(\omega_{nm}+\omega_{kl})t} \ , \tag{62}$$

where $n,m,l,k\in\{1,2\}$. Conservation of energy requires $\omega_{nm}+\omega_{kl}=0$, which can be satisfied by $k=m$ and $l=m$. Hence expression (62) reduces to

$$\dot{C}_k=-\frac{2\alpha}{3}\sum_{l=1}^{2}C_k|C_l|^2\omega_{kl}^3|\mathbf{x}_{kl}|^2 \tag{63}$$

or

$$\dot{C}_1=+AC_1|C_2|^2 \ , \tag{64a}$$

$$\dot{C}_2=-AC_2|C_1|^2 \ , \tag{64b}$$

where we have defined A as before in Eq. (55). Multiplying (64a) through by C_1^*, and the complex conjugate of (64a) by C_1 and adding these results give precisely the same nonlinear equation found in (54a). A similar calculation using C_2 gives (54b). Hence the $|C_i|^2$ obey the same time evolution equations (56) as before, exhibiting the chirruped hyperbolic profile, which is the trademark of the neoclassical account of the dynamics of spontaneous emission. This decay is not physical, but rather a result of the invalid application of the superposition principle that is assumed in expansion (41).

We now recalculate the spontaneous-emission decay rate in a different fashion, still within the context of neoclassical theory, and show that the theory does admit a correct exponential decay, so long as the expansion (41) is not used. Let us begin with the perturbed, time dependent Schrödinger equation (57). Instead of using the expansion (41), now we use the Fourier expansion (29). With this decomposition of the wave function, the perturbing radiation reaction potential V' of (57) becomes

$$V'(\mathbf{x},t)=\frac{2i\alpha}{3}\sum_{n,m}(\mathbf{x}\cdot\mathbf{x}_{nm})\omega_{nm}^3e^{i\omega_{nm}t} \ . \tag{65}$$

Notice the absence of the $C_i(t)$ here, as compared to Eq. (61). Ideally, the matrix elements $\mathbf{x}_{nm}$ are taken with respect to the exact solutions ψ_n to the entire nonlinear equation (57). However, we may as before substitute the approximate Coulombic solutions ψ_n^0 so as to iterate the first-order energy correction to E_n^0. Suppose we have prepared a state ψ_n that is an exact solution which minimizes the entire exact nonlinear action W. Let us assume that this state can be written $\psi_n=\psi_n^0+\delta\psi$ with energy $E_n=E_n^0+\delta\psi$. Then the Schrödinger equation

$$H\psi_n=i\frac{\partial\psi_n}{\partial t}=E_n\psi_n$$

can be written as

$$(H_0+H')(\psi_n^0+\delta\psi_n)=(E_n^0+\delta E_n)(\psi_n^0+\delta\psi_n)\ , \quad (66)$$

which reduces to

$$V'|n\rangle=-\delta E_n|n\rangle e^{-iE_nt}\ , \quad (67)$$

where we have used the separation $\psi_n^0(\mathbf{x},t)=\psi_n^0(\mathbf{x})e^{-iE_n^0t}$. We have neglected $\delta\psi_n$, and here $|n\rangle\equiv\psi_n^0(\mathbf{x})$. Inserting expansion (65) for V', operating from the left with $\langle k|$, and performing an additional sum on both sides with respect to a dummy summation index, we arrive at

$$\delta E_k=-\frac{2i\alpha}{3}\sum_{l,n,m}\omega_{nm}^3(\mathbf{x}_{kl}\cdot\mathbf{x}_{nm})e^{i(\omega_{nm}+\omega_{kl})t}\ . \quad (68)$$

Now we may integrate both sides over all time t, and then divide by 2π to extract an energy shift of the kth level, as per the method of (37), to be

$$\delta E_k=-\frac{2i\alpha}{3}\sum_{l,n,m}\omega_{nm}^3(\mathbf{x}_{kl}\cdot\mathbf{x}_{nm})\delta(\omega_{nm}+\omega_{kl})\ . \quad (69)$$

The δ function expresses a conservation of energy condition, which can be satisfied by the choice $k=m$ and $l=n$. [See Eqs. (32).] This finally yields an imaginary energy shift given

$$\delta E_k=-\frac{2i\alpha}{3}\sum_l\omega_{lk}^3|\mathbf{x}_{lk}|^2\ , \quad (70)$$

which can be rewritten as

$$\delta E_1=-\frac{2i\alpha}{3}\omega_0^3|\mathbf{x}_{21}|^2=-iA\ , \quad (71a)$$

$$\delta E_2=-\frac{2i\alpha}{3}\omega_0^3|\mathbf{x}_{21}|^2=+iA\ , \quad (71b)$$

which, when inserted back into Eq. (66), gives

$$\psi_1(\mathbf{x},t)=\psi_1(\mathbf{x})e^{-iE_n^0t+At}\ , \quad (72a)$$

$$\psi_2(\mathbf{x},t)=\psi_2(\mathbf{x})e^{-iE_n^0t-At}\ . \quad (72b)$$

Hence the excited level shows the correct exponential decay, but the ground state exhibits a clearly nonphysical exponential growth. Intuitively, this is because, at the somewhat primitive level of neoclassical theory, the radiation reaction perturbation V' is just as likely to perturb the ground state as it is the excited state. It is tempting to compare (72b) with the so-called *runaway solutions* of classical radiation reaction theory. This problem of the decay of the ground state did not arise in the complete self-field treatment given earlier. In the self-field expression (36) spontaneous emission emerges as the residue of a contour integral. For the ground state there is no pole enclosed by the contour, and hence the residue is zero, and the ground state is stable.

VII. CONCLUSION

In this paper we discussed the self-field approach to QED, emphasizing the theory's origin in the action at a distance theory of classical electrodynamics of Wheeler and Feynman. We showed how the theory could be used to give a nonrelativistic account of spontaneous emission and the Lamb shift in a two-level atom, with the usual exponential decay profile found in the standard approach.

We then showed how misuse of the superposition principle could lead to an incorrect prediction of a chirruped decay profile, as predicted in the neoclassical theory of Crisp and Jaynes. Reviewing the neoclassical theory, we showed that the chirruped decay could be eliminated by avoiding the use of the superposition principle, and that an excited two-level atom decays in the proper exponential fashion. The neoclassical theory appears to predict a runaway solution for a gound-state electron. We believe that this is due to the rather *ad hoc* fashion in which the neoclassical theory accounts for the radiation reaction field.

The self-field approach apparently contains elements of the neoclassical theory, but is more comprehensive in scope. The covariant elimination of the self-field A_μ^{self} of the electron through use of an EM Green's function takes into consideration the electron's radiation reaction field in a compelling and natural manner. In addition, self-field QED can make predictions about the Lamb shift, vacuum polarization, $g-2$, etc.—all of which seem to be beyond the scope of the original neoclassical theory. The theory of the general covariant self-field QED is given elsewhere.[15]

ACKNOWLEDGMENTS

We would like to thank M. O. Scully ($a=2$ and $b=1$), J. H. Eberly ($a=1$ and $b=2$), H. Walther, and W. Schleich for their important suggestions and comments which contributed to the development of this work.

[1]A. O. Barut, *Electrodynamics and Classical Theory of Fields and Particles* (Dover, New York, 1980), pp. 148–164.

[2]J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys. **17**, 157 (1945).

[3]H. Tetrode, Z. Phys. **10**, 317 (1922).

[4]G. Süssman, Z. Phys. **131**, 629 (1952).

[5]A. O. Barut and J. F. van Huele, Phys. Rev. A **32**, 3187 (1985).

[6]A. O. Barut and J. Kraus, Found. Phys. **13**, 189 (1983); A. O. Barut and N. Ünal, Physica **142A**, 467 (1987); **142A**, 488 (1987); A. O. Barut and Y. I. Salamin, Phys. Rev. A **37**, 2284 (1988); A. O. Barut, J. P. Dowling, and J. F. van Huele, *ibid.* **38**, 4405 (1988); A. O. Barut and J. P. Dowling, Z. Naturwiss **44a**, 1051 (1989).

[7]A. O. Barut and J. P. Dowling, Phys. Rev. A **36**, 649 (1987); **36**,

2550 (1987); Phys. Rev. A (to be published).

[8]M. D. Crisp and E. T. Jaynes, Phys. Rev. **179**, 1253 (1969).

[9]J. D. Jackson, *Classical Electrodynamics,* 2nd ed. (Wiley, New York, 1975).

[10]E. Schrödinger, Ann. Phys. (Leipzig) **82**, 265 (1926).

[11]A. O. Barut, *The Theory of the Scattering Matrix* (Macmillan, New York, 1968).

[12]H. A. Bethe, Phys. Rev. **72**, 339 (1947).

[13]R. Louden, *The Quantum Theory of Light* (Oxford University Press, Oxford, 1973).

[14]E. Fermi, Rend. Lincei. **5**, 795 (1927).

[15]*New Frontiers in Quantum Electrodynamics and Quantum Optics,* edited by A. O. Barut (Plenum, New York, 1990); A. O. Barut, Phys. Scr. **T21**, 18 (1988).